

Possible shunt compensation strategies based on Conservative Power Theory

Abstract. Considering the Conservative Power Theory (CPT), this paper proposes some novel compensation strategies for shunt passive or active devices. The CPT current decompositions result in several current terms (associated with specific physical phenomena), which were used for the definition of different selective current compensators, in terms of minimizing particular disturbing effects. Simulation results have been demonstrated in order to validate the possibilities and performance of the proposed strategies for single and three-phase four-wired circuits.

Streszczenie. W artykule zaproponowano kilka nowych metod kompensacji, opartych na Zachowawczej Teorii Mocy (CPT), pasywnymi i aktywnymi kompensatorami, włączonymi równolegle do odbiornika. Oparty na CPT rozkład prądu ujawnia szereg składowych (stowarzyszonych ze zjawiskami fizycznymi), które mogą być użyte do selektywnego określenia prądu kompensatora. Metody te i ich skuteczność zostały potwierdzone wynikami modelowania obwodów jednofazowych oraz obwodów trójfazowych, czteroprzewodowych. (Możliwe strategie bocznikowej kompensacji bazującej na teorii CPT).

Keywords: active power filters; harmonic compensation; selective compensation; unbalance compensation.

Słowa kluczowe: aktywne filtry mocy, kompensacja harmonicznych, kompensacja selektywna, kompensacja nierównoważenia.

Introduction

For the purpose of harmonic, reactive and unbalanced currents compensation, many authors have been working on the proposal of new methodologies. The goals are mostly related to the improvement of the compensation results or even to the simplification of passive and active power filter's design. Probably, the mainly addressed methods are based on the $p-q$ Theory or any of its adaptations [1-4]. However, it is very important to point out that there are many other alternatives, which can be, theoretically, even more flexible than the previous one and can make easier the understanding of physical phenomena and the potential of selective compensation [5, 6].

Therefore, this paper proposes using the Conservative Power Theory (CPT), proposed in [7-10], as a novel and alternative way to the design and implementation of shunt passive, active and hybrid power filters. The CPT current decompositions can be applied either to the definition of active filter control references, as well as for designing the filter components (switches, inductors, capacitors, etc.). Further, the compensation strategies can be very flexible, since the decompositions enable selective identification (and minimization) of different disturbing effects (harmonics, unbalances, reactive power).

In a recent paper [11], the authors discuss the utilization of the CPT for passive and active compensation of single and three-phase three-wire circuits. In this paper, the compensation of four-wire nonlinear and unbalanced circuits will also be considered.

Overview of the CPT framework

Proposed by Tenti *et al.* [7], the Conservative Power Theory has been recently revised and modified to deal with particular circumstances, on which the supply frequency could vary in time, such those related to smart grid applications [10].

The conservative power quantities defined by the CPT are based on the application of voltage and current Kirchhoff's Laws, as well as the Tellegen's theorem. For these reasons, such definitions are valid for any generic poly-phase circuit, with or without return conductor, independently of the voltage and current waveforms.

Additionally, a current decomposition methodology has been proposed, on which every current term is related to a specific physical phenomenon (power consumption P , reactive energy W , non linearities V and unbalances N).

Thus, among several possibilities, the last publications have shown that the CPT can be used in very interesting ways to control electronic power processors (EPP), such those used in active or hybrid power filters or also those related to distributed generation and or distributed compensation purposes [8,9] or also to design and control local compensators [11].

In order to avoid being repetitive in terms of overview, the authors refer to the original papers for additional details, particularly [10]; however, some basic definitions are restated in the following, under the assumption of periodic conditions. Note that bold variables indicate multidimensional vector representation and the index " μ " indicates per phase variables.

The instantaneous power was defined as in (1):

$$(1) \quad p = \mathbf{v} \circ \mathbf{i}$$

and the instantaneous reactive energy was defined as:

$$(2) \quad w = \hat{\mathbf{v}} \circ \mathbf{i}$$

where $\hat{\mathbf{v}}$ was defined as the vector containing the unbiased time integral of the phase voltages. Assuming phase variables, this quantity is calculated by the difference between the time integral and its average value, as indicated in (3):

$$(3) \quad \hat{v}_\mu = v_j - \bar{v}_j$$

Considering the average values of (1) and (2), the active power (P) and reactive energy (W) were defined respectively, as following:

$$(4) \quad P = \bar{p} = \langle \mathbf{v}, \mathbf{i} \rangle = \frac{1}{T} \int_0^T \mathbf{v}(t) \circ \mathbf{i}(t) dt$$

$$(5) \quad W = \bar{w} = \langle \hat{\mathbf{v}}, \mathbf{i} \rangle = \frac{1}{T} \int_0^T \hat{\mathbf{v}}(t) \circ \mathbf{i}(t) dt$$

The active power represents the conveyed average power. This definition is identical to the one of the conventional active power, but the reactive energy is a new definition that represents the average stored energy in the network, under generic conditions (including waveform distortions, unbalances, etc.).

It is important to underline that under sinusoidal conditions the term P coincides at any time with usual active power ($UI\cos\varphi$), while the reactive energy is related to the traditional reactive power by means of the fundamental frequency (ω), it means $W=Q/\omega=UI\sin\varphi/\omega$.

Based on the above definitions the phase currents were decomposed as:

Balanced active currents, which are the minimum phase currents (i.e., with minimum rms value) conveying active power P to the load:

$$(6) \quad i_{a\mu}^b = \frac{P}{\|v\|^2} \cdot v_{\mu} = G^b \cdot v_{\mu}$$

Balanced reactive currents, which are the minimum phase currents transferring reactive energy W , and they are related to the average energy being exchanged through the circuit:

$$(7) \quad i_{r\mu}^b = \frac{W}{\|\hat{v}\|^2} \cdot \hat{v}_{\mu} = B^b \cdot \hat{v}_{\mu}$$

Thus, either the active or the reactive currents have an explicit physical meaning. They are associated with the occurrence of the active power and reactive energy and they are related to the *load equivalent* conductance G^b , and susceptance, B^b , which hold for all phases.

Considering possible unbalanced systems, the CPT proposes the identification of **unbalanced active** ($i_{a\mu}^u$) and **reactive** ($i_{r\mu}^u$) **current** components by:

$$(8) \quad i_{u\mu} = (G^{\mu} - G^b) v_{\mu} + (B^{\mu} - B^b) \hat{v}_{\mu}$$

$$i_{u\mu} = i_{a\mu}^u + i_{r\mu}^u$$

where G^{μ} e B^{μ} are the conductance and susceptance of each phase and they are given by:

$$(9) \quad G^{\mu} = \frac{P_{\mu}}{\|v_{\mu}\|^2}, \quad B^{\mu} = \frac{W_{\mu}}{\|\hat{v}_{\mu}\|^2}$$

The **void currents**, which are the remaining phase currents:

$$(10) \quad i_{v\mu} = i_{\mu} - i_{a\mu}^b - i_{a\mu}^u - i_{r\mu}^b - i_{r\mu}^u$$

By definition, all current terms are orthogonal [10]:

$$(11) \quad \|i\|^2 = \|i_a^b\|^2 + \|i_a^u\|^2 + \|i_r^b\|^2 + \|i_r^u\|^2 + \|i_v\|^2$$

Therefore, the apparent power has been defined as:

$$(12) \quad A = \|i\| \|v\| = \sqrt{P^2 + Q^2 + N_a^2 + N_r^2 + V^2}$$

$$= \|i_a^b\| \|v\| + \|i_r^b\| \|v\| + \|i_a^u\| \|v\| + \|i_r^u\| \|v\| + \|i_v\| \|v\|$$

where: P is the active power, Q is the reactive power, N is the unbalance power (divided into active and reactive parts) and V is the void power. It is worth mentioning that, differently from (4) and (5), such apparent power (A) is a non conservative quantity.

Notice from (12) that the unbalance power was divided into *unbalanced active power* (N_a) and *unbalanced reactive power* (N_r), which are respectively related to possible

conductance and susceptances imbalances. For single-phase circuits, these power components result null.

Considering the novel apparent power definition (12), it has been defined in (13) a **total power factor**, which is not only affected by the reactive power, but also by the unbalance and void power components:

$$(13) \quad \lambda = P / A$$

Further discussions about such currents and related power components can be found in [7] to [10]. However is already possible to notice that some of the parcels of (11) could be useful to power conditioning purposes. For example, one could use the unbalance or void current components to minimize, selectively or not, the load current unbalances or distortion. The same could be done in terms of the reactive component.

Selective compensation strategies

Considering possible reactive, harmonic and unbalance compensators, two important decisions should be done: the compensation strategy [4-6,12-14] and the compensator topology [15].

This paper deals particularly with possible compensation strategies. In such case, several different possibilities can be defined by means of the CPT current and power decompositions. The disturbing current components (reactive, unbalanced or void), are represented by $i_{r\mu}^*$, $i_{u\mu}^*$, $i_{v\mu}^*$ or any of their associations, e.g., $(i_{r\mu}^* + i_{v\mu}^*)$. Note that these current components can be selectively compensated by means of different compensators, as foreseen in the scheme of Fig. 1.

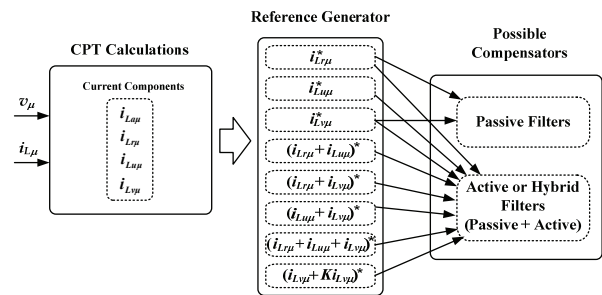


Fig.1. Selective compensation scheme based on the CPT

Assuming the utilization of electronic compensators (active power filters), any of the current components of Fig. 1 could be applied in the control strategy in order to minimize or even eliminate the undesired disturbing effects. However, although the enormous advances on power electronics, this kind of application still yields in quite expensive solutions. Thus, it should be very useful if the compensation strategy were able to provide the accurate current reference in order to ensure that specific disturbing components achieve standardized values, with minimum efforts. Thus, if one is interested in compensating just the reactive current components, a possible formulation should be:

$$(14) \quad i_{r(comp)}^b = i_{r(load)}^b - i_{a(load)}^b \sqrt{\frac{1}{PF(\lim)} - 1}$$

where $i_{r(comp)}^b$ is the parcel of balanced reactive current that should be compensated in order to ensure a resulting power factor (traditional) approximately equal to $PF = P / \sqrt{P^2 + Q^2}$. This could be done by means of a

permanent or switched passive compensator (usually capacitors).

The average energy in the capacitor should be [10]:

$$(15) \quad W_{C\mu} = -C \cdot \|v_{\mu}\|^2$$

and assuming low distorted supplying voltages, the reactive power would be derived by:

$$(16) \quad Q_C = -\omega \cdot W_{C\mu} = -\omega \cdot C \cdot \|v_{\mu}\|^2 = \|v_{\mu}\| \cdot \|i_{r\mu}^b(comp)\|$$

Therefore, the capacitor value should be determined by:

$$(17) \quad C = \frac{\|i_{r\mu}^b(comp)\|}{\omega \cdot \|v_{\mu}\|}$$

Of course, if an active power filter is available, the overall reactive current or i_r^b could be minimized by means of electronic compensation. In addition, if one is interested in minimizing the reactive power and also some specific harmonic frequency (ω_h), a passive LC filter could be designed using the same capacitor (17) and $L=1/\omega_h^2 C$.

Considering the current distortion, a *void distortion factor* could be defined as:

$$(18) \quad \sigma_V = \frac{\|i_{v\mu}\|}{\|i_{\mu}\|} \approx THD_i$$

Assuming the goal of limiting the current distortion in order to ensure that the compensated system results under certain limits ($\sigma_V(lim)$), the compensator should be able to eliminate part of the void current $i_{v(comp)}$:

$$(19) \quad \begin{aligned} \sigma_V(load) - \sigma_V(comp) &= \sigma_V(lim) \\ \frac{\|i_{v\mu}\|}{\|i_{\mu}\|} - \frac{\|i_{v\mu}(comp)\|}{\|i_{\mu}\|} &= \sigma_V(lim) \\ \|i_{v\mu}(comp)\| &= \|i_{v\mu}\| - \|i_{\mu}\| \sigma_V(lim) \end{aligned}$$

Such current component could be compensated based on active power filter or either based on associations of particular LC filters. Of course, if it were necessary the compensation of the total void current, it is also possible, however it will clearly results in more sophisticated and expensive solutions.

It is also possible the definition of hybrid compensator, as indicated, e.g., by the overall structure of Fig. 2. In such cases, the reactive and void current compensation are split in terms of the passive and active filters. It can be done designing the active filter in order to complete the compensation provided for the passive compensators.

In the sense of selective compensation, other power quality factors could be defined and associated to particular current components, which should be minimized or reduced to improve, e.g., the total power factor (λ) defined in (13).

In order to evaluate the CPT application for selective compensation purposes, next sections illustrate its use on different topologies of single and three-phase four wire compensators.

Simulation results

A. Single-phase circuit - nonlinear load

Considering a single-phase system, Fig. 2 shows the simulated power circuit, while Table 1 represents the values of the grid voltage, line impedances and load parameters. Fig. 3 shows the measured PCC current before the compensation. Note that the current is significantly distorted.

Considering both compensators of Fig. 2, Table 2 indicates the main parameters of each compensator (passive and active). The LC filters were tuned on 175 Hz and 295 Hz. The quality factors were adjusted to 30 and 25. The capacitors were chosen so that the traditional power factor (PF) results to 1 ($I_{r,RMS} \approx 0$) after compensation.

The active filter was based on a (voltage source) three level PWM converter, with sampling and switching frequency of 12 kHz and 400 V DC link voltage. Both DC bus and output current controllers were based on proportional-integral (PI) regulators, with 5 Hz and 2 kHz bandwidth, respectively.

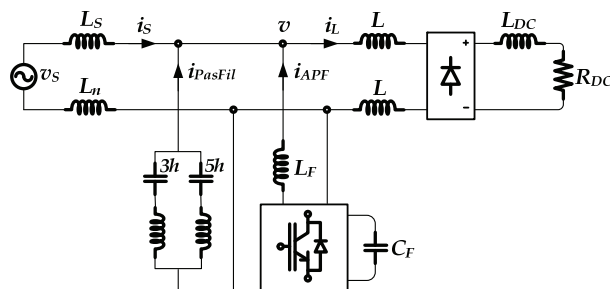


Fig. 2. Single-phase power circuit – non linear load (harmonic current source type)

Table 1. Source voltage, load and line impedances of the single-phase power system

Source	Line	Load
$V_s=127 \angle 0^\circ$ V _{rms}	$R_s=10$ m Ω $L_s=0.1$ mH	$L_D=5$ mH
60 Hz	$R_n=10$ m Ω $L_n=0.1$ mH	$R_{DC}=2 \Omega$

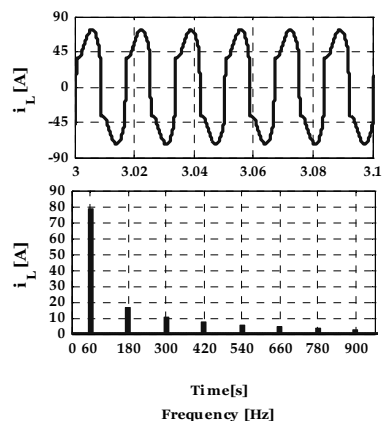


Fig. 3. Load current waveform and spectrum

Table 2. Parameters of the power conditioners

Passive filters		Active power Filter
$R_3=0.1777 \Omega$	$R_5=0.1265 \Omega$	$R_F=0.013 \Omega$
$L_3=4.800$ mH	$L_5=1.700$ mH	$L_F=0.35$ mH
$C_3=170.57$ μ F	$C_5=170.57$ μ F	$C_F=3500$ μ F

A.1) *Passive filter compensation:* Fig. 4 shows the compensation results considering just the passive LC filters. Note that the source current distortion is reduced. However,

it is not sinusoidal, since just the 3rd and 5th harmonics were compensated.

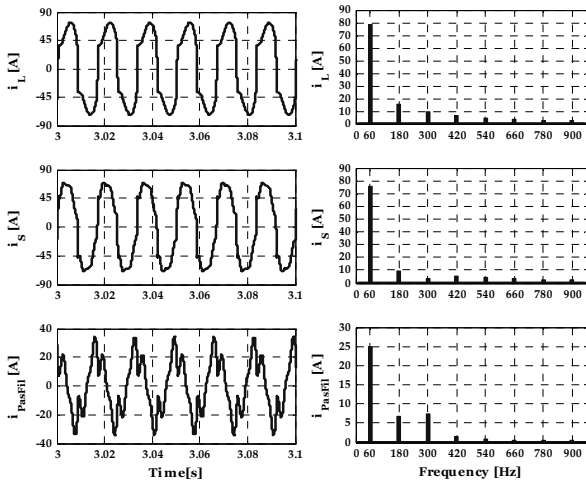


Fig. 4. Load current (i_L), source current (i_S) and passive filter current (i_{PasFil}) (waveforms and spectra) – A.1

A.2) **Active filter compensation:** Fig. 5 shows the compensation results for both, reactive and void current. In this case the compensation is almost complete.

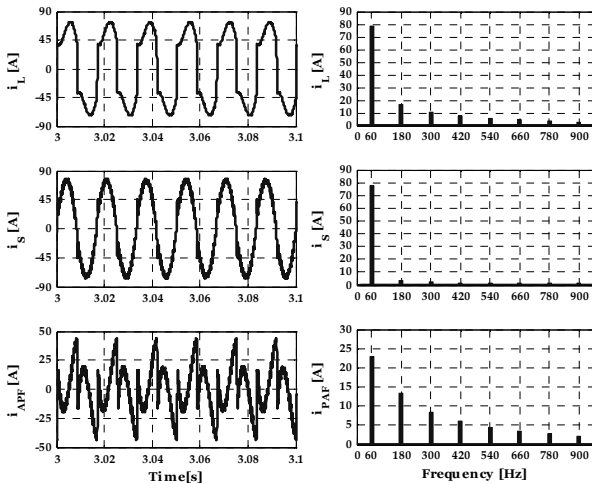


Fig. 5. Load current (i_L), source current (i_S) and active filter current (i_{APF}) (waveforms and spectra) – A.2

A.3) **Hybrid filter compensation:** Fig. 6 shows the compensation results. In this case the compensation was split between the two compensators (passive and active). The passive filter compensates about 36% of the current distortion. Thus, the active filter was set to compensate 50% of the third harmonic and the 0% of the fifth harmonic in the void current $\{i_{ref}^* = i_v - 0.5(i_{v3h}) - (i_{v5h})\}$. Note that the compensation behavior yields practically the same as in A.2, but in this case the active filter RMS and peak current values are significantly lower. In that case the active power filter was set to minimize the total void current. Such information and the costs associated to one or other compensator topology may define the final compensation scheme.

B. Three-phase four-wire circuit – nonlinear and unbalanced load (neglected line impedance)

Considering a three-phase four-wire non linear and unbalanced circuit, as indicated in Fig. 7, a four-leg active filter has been applied in order to demonstrate the possibility of compensating not only the reactive and harmonic currents, but also possible load unbalances.

Details of source voltages, as well as the load and active filter parameters can be found in Table 3.

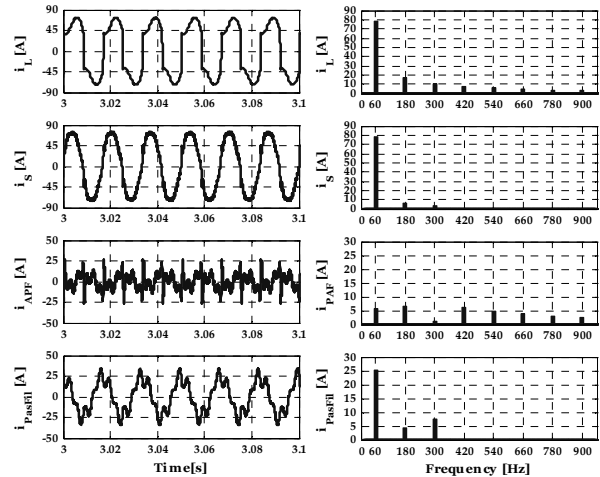


Fig. 6. Load current (i_L), source current (i_S), active filter current (i_{APF}) and passive filter current (i_{PasFil}) (waveforms and spectra) – A.3

In such case, the active filter was based on a three-phase four-leg PWM converter (voltage source type), with sampling and switching frequency of 12.6 kHz and 400 V DC link voltage. For simplicity, the DC bus voltage was maintained by means of a DC voltage source. The filter current controller was based on proportional-integral (PI) regulators, with 2 kHz bandwidth and phase margin of 60°.

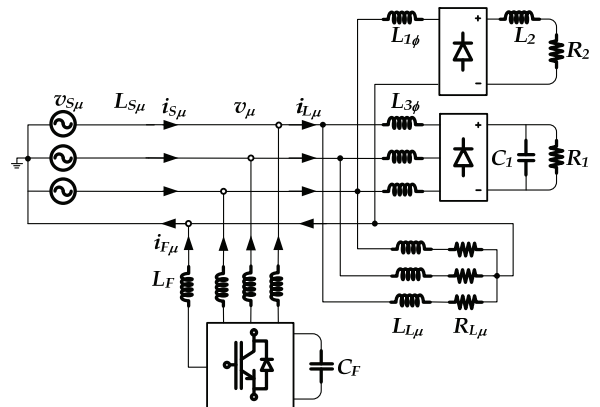


Fig. 7. Three-phase four-wire circuit with non linear and unbalanced load

Table 3. Parameters of the source voltage, load and active filter

Source (60 Hz)	Load Parameters	
$v_{Sa} = 127 \angle 0^\circ \text{ V}$	$R_{La} = 2 \Omega,$ $L_{La} = 47 \text{ mH}$	$L_{3\phi} = 80 \text{ uH},$ $C_1 = 2 \text{ mF}, R_1 = 5 \Omega$
$v_{Sb} = 127 \angle -120^\circ \text{ V}$	$R_{Lb} = 2 \Omega,$ $L_{Lb} = 47 \text{ mH}$	$L_{1\phi} = 100 \text{ uH},$ $L_2 = 5 \text{ mH}, R_2 = 7 \Omega$
$v_{Sc} = 127 \angle 120^\circ \text{ V}$	$R_{Lc} = 1 \Omega,$ $L_{Lc} = 24.7 \text{ mH}$	
Active filter		
$R_F = 1.8 \text{ m}\Omega; L_F = 520 \text{ uH}; C_F = 3500 \text{ uF}$		

B.1) **Total non active compensation** – in this case the objective is to compensate all the undesirable current components of the load (reactive, unbalances and harmonics). The resulting supply currents are shown in Figs. 8 and 9. The compensated currents are practically sinusoidal, balanced and in phase with the voltages, such as in the case of an equivalent balanced resistive load, since the non active currents ($i_F^* = i_v + i_u + i_w$) have been compensated. Note that the current through the return

conductor has been practically eliminated (i_{source_n}). In this case, the active filter reference for the return conductor controller was set to $-(i_{Fa}^* + i_{Fb}^* + i_{Fc}^*)$.

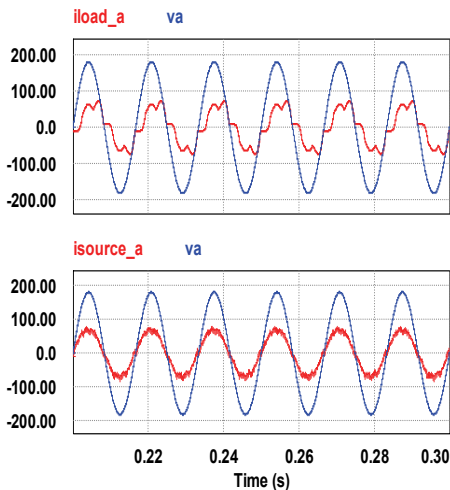


Fig. 8. Source voltage (phase a), load currents (i_{load_a}) and source currents (i_{source_a}): non active currents compensation – B.1

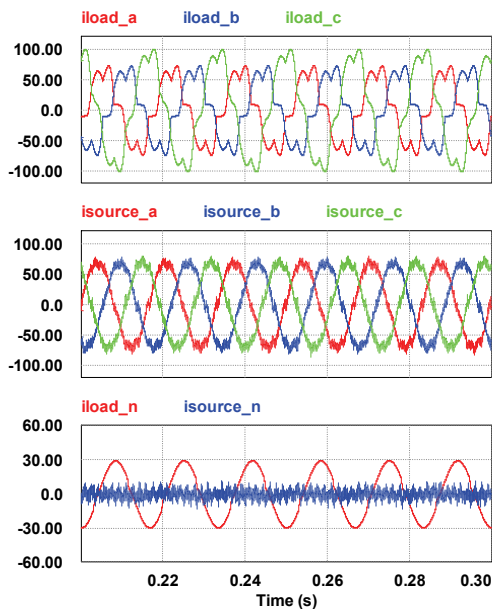


Fig. 9. Load currents (i_{load}), source currents (i_{source}) and return conductor currents: non active currents compensation – B.1

B.2) *Unbalance compensation* – in this case the goal is to compensate only the unbalanced current components (8). Figs. 10 and 11 show the compensation results. Note that the currents remain distorted, but they are practically balanced. In addition, the return conductor current has been compensated, as well as in the previous case.

B.3) *Void current compensation* – Figs. 12 and 13 show the case on which the active filter was set to compensate just the void current components (10). Now the resulting compensated currents are practically sinusoidal, but they are not balanced, neither in phase with the voltages. As expected, in this case, the current through the return conductor is practically not affected by the compensation.

B.4) *Reactive currents compensation* – From Figs. 14 and 15, it is possible to notice the results for balanced reactive current compensation (5). Observe that the resulting compensated currents are non sinusoidal and

unbalanced, since the void and unbalanced currents were not applied in the active filter control. Again, the return conductor current was not compensated in this case.

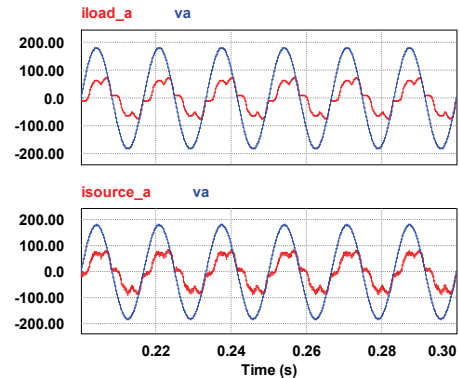


Fig. 10. Source voltage (phase a), load currents (i_{load_a}) and source currents (i_{source_a}): unbalanced currents compensation – B.2

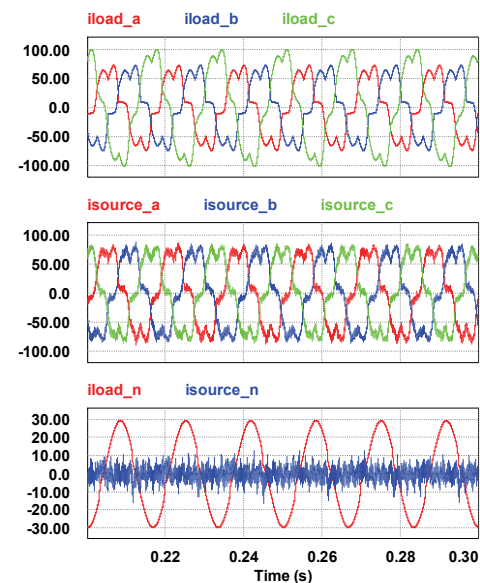


Fig. 11. Load currents (i_{load}), source currents (i_{source}) and return conductor currents: unbalanced currents compensation – B.2

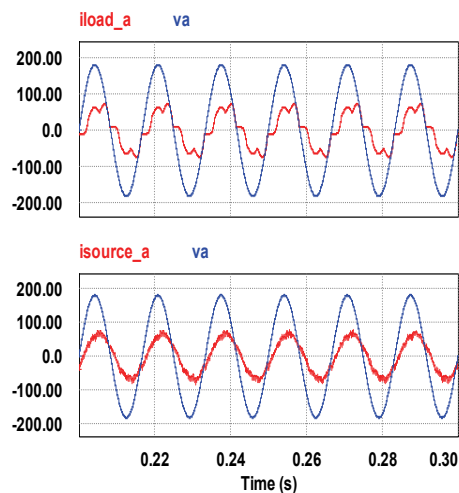


Fig. 12. Source voltage (phase a), load currents (i_{load_a}) and source currents (i_{source_a}): void currents compensation – B.3

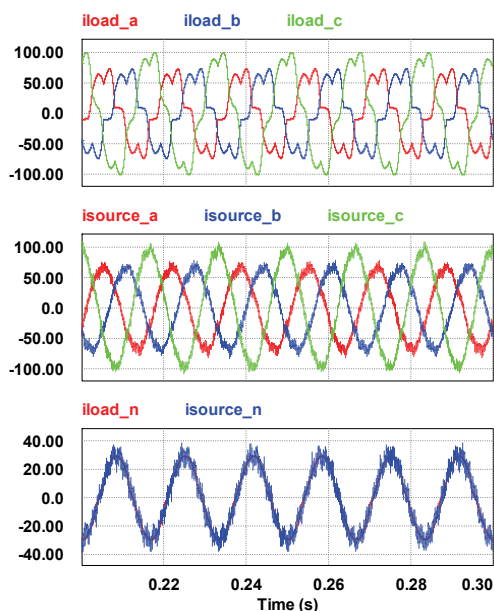


Fig. 13. Load currents (i_{load}), source currents (i_{source}) and return conductor currents: void currents compensation – B.3

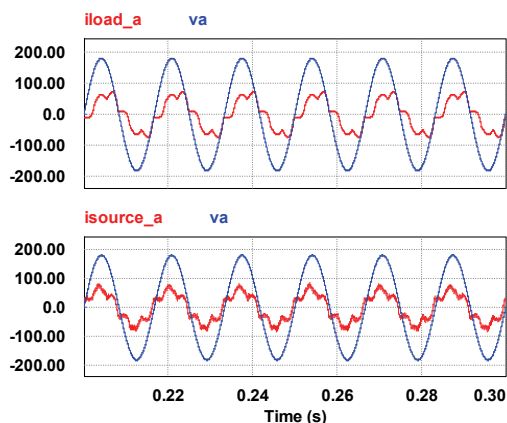


Fig. 14. Source voltage (phase a), load currents (i_{load_a}) and source currents (i_{source_a}): reactive currents compensation – B.4

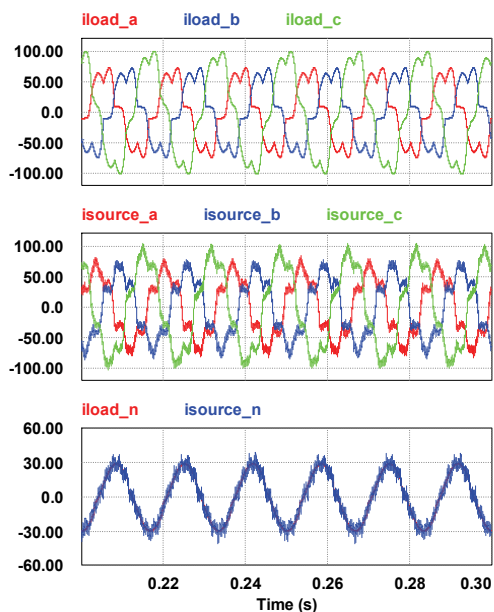


Fig. 15. Load currents (i_{load}), source currents (i_{source}) and return conductor currents: reactive currents compensation – B.4

Conclusions

This paper has demonstrated some of the possible compensation strategies based on the Conservative Power Theory. As verified, different passive, active or hybrid compensators could be designed or controlled by means of such proposal. It is important to notice the flexibility of the methodology, which allows the designer to chose among one or more disturbing effects to be minimized (reactive, unbalance or void current) or also the percentage of the compensation among them (based for example, on specific standard limits) or among the compensator's topologies.

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