

## Novel S-transform information fusion for filtering ultrasonic pulse-echo signals

**Abstract.** Direct evaluation of ultrasonic signals requires data analyses with an acceptable level of noise. Ultrasonic signals represent a specific category of time domain signals to be analyzed. In order to increase a difference between the level of noise and the amplitude of the ultrasonic pulse a suitable method for signal filtering has to be used. Within this article we discuss and evaluate a novel signal denoising method. The S-transform for signal analysis and processing was used. This transformation has been recently introduced for ultrasonic echo analyses. Proposed transformation represents an intermediate stage between the Fourier transform analysis and the wavelet transform analysis. In order to filter ultrasonic signals from the Electromagnetic Acoustic Transducer (EMAT) with a high level of noise, new, different approach in signal filtering was developed based on an information fusion. Suggested method is able to process the pulse-echo signal in its full complexity. Proposed method offers good results in studied ultrasonic signals in comparison to digital filter or wavelet denoising.

**Streszczenie.** Bezpośrednia ocena sygnału ultradźwiękowego wymaga analizy danych obciążonych szumami. W celu zwiększenia różnicy między amplitudą sygnału a szumami użyto specjalnej metody filtrowania. Zastosowano transformację S do analizy ultradźwiękowego sygnału echa. Tę transformację jest metodą pośrednią między transformacją Fouriera a transformacją falkową. Użyto nowej metody bazującej na fuzji informacji. Testy potwierdziły że nowa metoda może być skuteczniejsza niż filtrowanie cyfrowe czy odszumianie falkowe (**Nowa metoda filtrowania sygnału ultradźwiękowego bazująca na transformacji S i fuzji informacji**)

**Keywords:** EMAT, ultrasonic signal, correlation, pulse-echo, signal filtering, information fusion.

**Słowa kluczowe:** sygnał ultradźwiękowy, filtrowanie sygnału.

### Introduction

A characteristic feature of ultrasonic waves is their frequency, which is higher than the hearing range of the human ear (typically 20 kHz). The upper range of these waves is up to 3 GHz, but usually the upper bound of the frequency rarely exceeds 20 MHz. The ultrasonic inspection of an examined specimen can be carried out in two ways: by active sensing and by passive sensing. For active sensing a transmitter and a receiver are used in the measurement setup. The transmitter excites the signal and the receiver receives it. If there is any damage in the volume of the specimen, then the ultrasonic signal is altered by the material inhomogeneity. Information about the material structure is gained from the analysis of a received signal. For a passive inspection no transmitters are used and only receivers are attached to the specimen.

There are several possibilities how to arrange transducers on the specimen. Common modes for the transmitter and receiver placement are pulse-echo mode, pitch-catch mode, and through-transmitter mode as it is showed in Fig. 1.

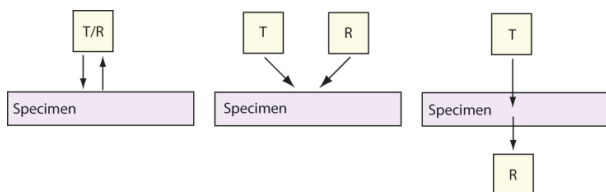


Fig. 1 Common modes of transmitter-receiver arrangements: pulse-echo, pitch-catch and through-transmitter mode

The received signal from an ultrasonic system can be displayed in the four different manners known as A-scan, B-scan, C-scan and S-scan also known as Sector-scan, see Fig. 2. A-scan is based on a full vibratory motion of the receiver which is a function of time for a specific location.

With B-scan the amplitude of received echoes is transformed in to the grey level. When speed of the movement is constant, the time record corresponds to the position of the probe. With known speed of sound inside the specimen, the arrival time of the echo is directly proportional to the depth of the reflecting boundary. Outer

stripes in the record represent the stronger signal reflected from the backwall. Thinner lines in Fig. 2 correspond to the local crack.

When the transducer is moved in a plane parallel to specimen surface and the peak value of the received signal is plotted as a function of the transducer position, then the generated image is called C-scan. The image is based on information acquired in the region between the initial echo and the backwall echo. The peak value is plotted as a function of position.

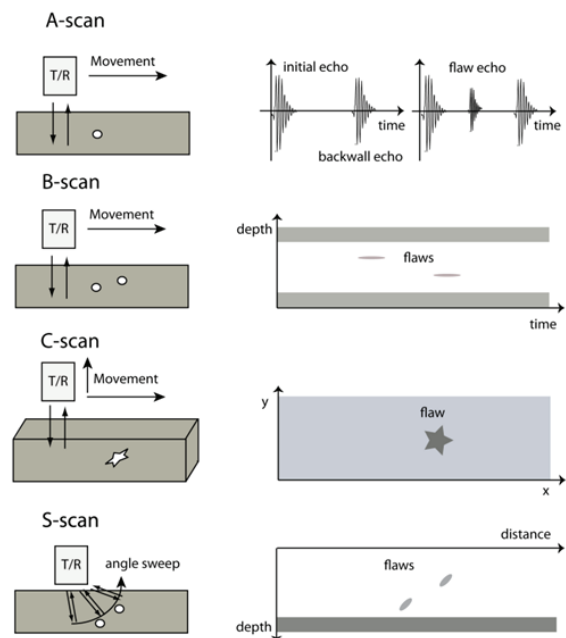


Fig. 2 Schematic of A-scan, B-scan, C-scan and S-scan

The transducer with selectable angle of the emission could be used to generate S-scan. This method offers virtual cut through specimen. While B-scan is a function of the arrival time of the echo and the record time, S-scan plots echoes values as a function of the received time and angle. The situation is shown in Fig. 2.

All typical ultrasonic visualizations used for the structure analysis are primarily based on A-Scan data. Within this scope signal processing methods for one dimensional time varying signals are needed.

The electromagnetic acoustic transducer

Still commonly used piezoelectric transducers brings widely accepted drawback of a direct mechanical coupling necessary for transmission the ultrasonic signal from transducer to the specimen and back. Whereas the Electromagnetic Acoustic Transducer (EMAT) links ultrasonic signal with the probe via changes in dynamic magnetic field and its geometrical configuration relative to the static magnetic field. The laboratory EMAT system is based on "bulk-wave" generation. The permanent magnet produces the biasing, static magnetic field orthogonal to the surface. A detailed description of the probe design is in [1].

There are two types of bulk waves, longitudinal waves and shear waves. When longitudinal waves propagate through an infinite medium only orthogonal stress is generated. The propagation of shear waves generates shear stress. Wave speed of these two wave types are described by:

$$(1) \quad c_L = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{E(1 - \nu)}{\rho(1 + \nu)(1 - 2\nu)}}$$

$$(2) \quad c_s = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{2\rho(1 + \nu)}}$$

where  $c_L$  is the longitudinal wave speed (m/s),  $c_s$  is the shear wave speed (m/s),  $\rho$  is the density ( $\text{kg/m}^3$ ),  $\lambda$  is the Lamé's first parameter ( $\text{kg/ms}^2$ ),  $\mu$  is the Lamé's second parameter or the shear modulus ( $\text{kg/ms}^2$ ),  $E$  is the Young's modulus ( $\text{kg/ms}^2$ ) and  $\nu$  is the Poisson's ratio (-).

Used system generates the longitudinal wave and the radially-polarized shear wave. The both waves are propagating in the straight direction at the same time, for which this is called dual-mode EMAT. If the metal has an orthorhombic elastic anisotropy, due to the texture, the shear wave decomposes into two polarizations along the two principal directions [2].

The transducers used for an ultrasonic measurement works in impulse stimulation. The impulse transducer operation leads to the wide bandwidth. There exists quality factor  $Q$  which describes mentioned width:

$$(3) \quad Q \cong \frac{f_r}{f_1 - f_2} = \frac{f_r}{B}$$

where  $\beta = \omega_r/2Q$  is the damping factor,  $A$  denotes gain and  $\omega_r$  denotes angular frequency  $f_r$ .

The  $f_r$  denotes the transducer central frequency,  $f_1$  and  $f_2$  are side frequencies where the amplitude drops about 3 dB in compare to the central frequency and  $B$  denotes the bandwidth [3].

Ultrasonic damped oscillations are expressed by

$$(4) \quad a = Ae^{-\beta t} \sin \omega_r t$$

For a pulse ultrasonic evaluation it is required a good resolution with an acceptable sensitivity. Equation 3 denotes the contradiction of those requirements [3]. The optimal signal processing transform has to follow the optimal model described by equation 4 with the discrimination of noise and the ultrasonic pulses.

Following paragraphs will describe wavelet and Fourier based transforms for time-frequency processing in order to increase the contrast within the ultrasonic signal.

## Signal filtering

The search for an optimal tool for suppressing embedded noise in the ultrasonic evaluation can be traced to Bilgutay [4]. Bilgutay et. al used set of digital filters to split a signal and they designed a non-linear technique to reconstruct filtered signal in the time domain. Hoess et. al used Wiener filter to process the signal in frequency domain [5]. After introduction of the wavelets [6], wavelet based signal processing improves the signal to noise ratio of the ultrasonic signal [7, 8]. There are two possible trends in the improvement of the ultrasonic measurement signal to noise ratio. The first one is an introduction of a novel mathematic transformation. The second one is an invention of a novel signal processing algorithm suitable for the ultrasonic application in connection with state-of-the-art transformation (S-Transformation, Wavelet Transformation etc.). So far, there is unexploited potential of these transformations. Therefore this article aims to develop an advanced signal processing algorithm based on currently used transformations.

## S-transformation

The Fourier Transform (FT) describes a signal in terms of complex sinusoids series, with a varying amplitude and phase. According to equation 4 ultrasonic pulsed signals can be described by sinusoids. The sinusoidal basis functions of the Fourier Transform are purely periodic and infinite in extent, and the FT entirely converts a signal between the time and frequency domain, with no direct temporal information remaining after the transform. In case of ultrasonic pulses it represents imperfect approximation in spite of general recognition of the FT analysis. To allow examination of non-stationary signals, a number of solutions have been proposed, including the Short-Time Fourier Transform (STFT) and more recently the S-Transform (ST). ST is an extension of the STFT which uses frequency-dependent scaling windows in an analogy to the wavelet transform. STFT can use any window function [9] but ST uses Gaussian window which achieves the optimal time and the frequency resolution. The ST of the time signal  $a(t)$  is defined in [10] as

$$(5) \quad S(\tau, \nu) = \int_{-\infty}^{+\infty} a(t) \frac{|v|}{\sqrt{2\pi}} e^{-\frac{(\tau-t)^2 v^2}{2}} e^{-2\pi\nu t} dt$$

where  $\tau$  and  $\nu$  are the S-transform time and frequency coordinates.

Equation 5 has the same form as FT equation, but adds a normalized-area Gaussian window for time localization. The  $\nu$  parameter causes decrease of the window width with the increasing frequency. This automatically adjust the ST window to provide a progressive trade-off between the time and frequency resolution for the each frequency, with an improved frequency resolution at low frequencies and better time resolution at high frequencies in compare to FT. Like FT, ST produces a complex spectrum that includes both the frequency and globally referenced phase information [11, 12].

## Wavelet transformation

Wavelets are functions that are used to represent temporal processes. Ultrasonic pulsed signals are usually time and frequency limited. For this reason, the utilization of time-frequency wavelet analysis was already evaluated [13]. If wavelet has properties of the compact or the approximate compact support in the time and frequency domain, it can be treated as a band-pass filter. A bank of band-pass filters called wavelet packets can be obtained

with signal processing on different central frequencies and bandwidths by compressing/dilating and shifting the mother wavelet.

Suppose that  $\varphi(t)$  is an arbitrary mother wavelet and its FT is  $\psi(\omega)$ . The central frequency and frequency resolution are expressed as

$$(6) \quad \omega_0 = \frac{\int_{-\infty}^{+\infty} \omega |\psi(\omega)|^2 d\omega}{\int_{-\infty}^{+\infty} |\psi(\omega)|^2 d\omega}$$

$$(7) \quad \Delta\omega = \sqrt{\frac{\int_{-\infty}^{+\infty} (\omega - \omega_0)^2 |\psi(\omega)|^2 d\omega}{\int_{-\infty}^{+\infty} |\psi(\omega)|^2 d\omega}}$$

where  $\omega = 2\pi f$  is the angular frequency,  $\omega_0$  is central frequency of the mother wavelet.

We can define a frequency window of the mother wavelet as  $\left[\omega_0 - \frac{\Delta\omega}{2}, \omega_0 + \frac{\Delta\omega}{2}\right]$  and with this range is defined band pass region.

### Echo detection

Let define a basic physical observation model that we wish to consider. The observed continuous-time waveform consists of two possible signals ultrasonic echoes and noise. Our objective is to decide which of the two possible signals is present, and we wish to do so by processing a finite number (say  $n$ ) of samples taken from the observed waveform. This problem can be modeled statistically by the following hypothesis pair for the observed space:

$$(8) \quad H_0: x_i = r_i \quad (i = 1, 2, \dots, n)$$

versus

$$(9) \quad H_1: x_i = s_i + r_i \quad (i = 1, 2, \dots, n)$$

where  $x = \{x_1, \dots, x_n\}$  is an observation vector consisting of samples from the observed waveform,  $s = \{s_1, \dots, s_n\}$  is a vector of samples from the possible signal of interest, and  $r = \{r_1, \dots, r_n\}$  is a vector of noise samples. We are actually trying to detect a signal embedded in noise. For this purpose of this treatment we will assume that noise is independent of the signal under the each hypothesis and that its probability distribution does not depend on which hypothesis is true. This assumption is valid if we assume that the noise part caused by the spurious signal reflection from material boundaries can be neglected.

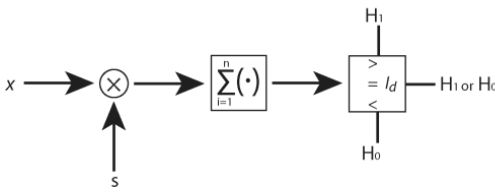


Fig. 3 Optimum detector for coherent signals and i.i.d. noise

Optimal procedure for deciding between  $H_0$  and  $H_1$  can be derived if we have models for the statistical behavior of the ultrasonic signal and noise. According to Harris [9] the signal  $s$  is classified as one of three basic types. The signal can be completely known (deterministic), it can be known except for a set of unknown (random)

parameters, or it can be completely random and thus specified only by their probability distributions. The more different the amplitude distribution from noise distribution is, the correct decision is made with a higher probability [14]. Suppose that the noise samples  $r_1, \dots, r_n$  are independent and identically distributed (i.i.d.) with marginal distribution  $N(0, \sigma^2)$  presented in Fig. 6. The structure depicted in Fig. 3 is known as a correlation detector or the correlator. This optimum detector can be viewed as a system that inputs the observation sequence  $x_1, \dots, x_n$  to a digital linear filter and then samples the output at time  $n$  for comparison to a threshold [15].

For a measured signal, where  $s_i$  is taken from the first backwall echo, a threshold level estimation can be taken as

$$(10) \quad l_d = 3\sigma_v$$

where  $\sigma_v$  denotes standard deviation of an echo detection signal. When a signal has normal distribution like white noise, three times standard deviation covers 99% of this signal. The threshold process described in Fig. 4 as binary thresholding (corresponds to a binary decision task) for echo detector is described as

$$(11) \quad v_d(i) = \begin{cases} 1, & |v(i)| > l_d \\ 0, & |v(i)| \leq l_d \end{cases}, (i = 1, 2, \dots, n)$$

### Information fusion

Our research work on ultrasonic signal filtering is focused on the echo extraction from the signal where the echo has the amplitude smaller than the noise level is. Previous benchmark of several common filtering methods produced insufficient results from flaw echo detection and evaluation point of view [16]. New concept is proposed in Fig. 4. This method merges information from ultrasonic echo detector and S-transform analysis.

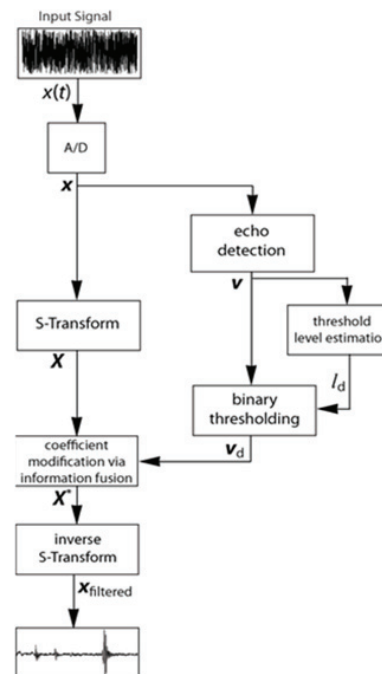


Fig. 4 A description of an information fusion idea used with S-Transform

Signal processing methods based on ST were studied for different applications [17, 18]. The difference between WT and ST is mainly in core function and transformation

algorithm characteristics. Core functions were already discussed in previous section. WT commonly used transformation algorithm is called Discrete Wavelet Transform (DWT). DWT incorporates convolution of the signal with low and high pass filters followed by downsampling [19]. Downsampling according to algorithm displayed in Fig. 4 introduces additional time localization uncertainty. This is the reason why ST is used because downsampling is not necessary. The second reason is that Fourier analysis is widely used and therefore could be easily implemented into digital signal processor. The above discussion led to a design of an information fusion algorithm based on echo detector and S-Transform.

A digital filter modifies the signal in the frequency domain by multiplication of the signal and the filter frequency characteristic. The information fusion algorithm modifies the signal in the ST domain by multiplication of the signal and correlation filter time domain characteristic. The S-domain adjusting process is expressed in hypothesis testing as:

$$(12) \quad H_1 \approx H_{detected} X(\tau_d, \nu) \rightarrow X^*(\tau_d, \nu) = X(\tau_d, \nu) * L_d$$

$$(13) \quad H_0 \approx H_{noise} X(\tau_n, \nu) \rightarrow X^*(\tau_n, \nu) = X(\tau_n, \nu) * L_n$$

where  $X(\tau, \nu)$  denotes ST coefficients described by the time  $\tau$  and the frequency location  $\nu$ .  $L_d$  denotes low pass filter for ultrasonic signal with high frequency noise component (FIR, order = 8,  $f_{cut-off}$  = 10 MHz) and  $L_n$  denotes low pass filter for noise (FIR, order = 8,  $f_{cut-off}$  = 10 MHz). The attenuation at cut-off frequencies is fixed at 6 dB (half the passband gain). A signal on a one frequency is called voice. The detector hypothesis is applied on the transformed signal by the multiplication with different digital filters in frequency domain. Adjusted coefficients are transformed back to the time domain via inverse ST. Multiple inverse ST algorithms were proposed [20, 21, 22]. We used according to Stockwell [20] the simplest and easily implementable version where all filtered voices are tot up and inverse Fourier transform is used.

## Results

The example of the measured and processed ultrasonic signal is presented in Fig. 5. The initial echo was cut off because its amplitude was out of analog to digital converter range. Histogram of the signal is in Fig. 6. From the histogram can be deduced a significant noise part with Gaussian distribution. The signal decomposition with ST is in Fig. 7. The second information source for the fusion, the echo detection is placed above the ST analysis. As it was mentioned in previous section a reference signal for the echo detection was used the first backwall echo signal marked with ping box in Fig. 5.

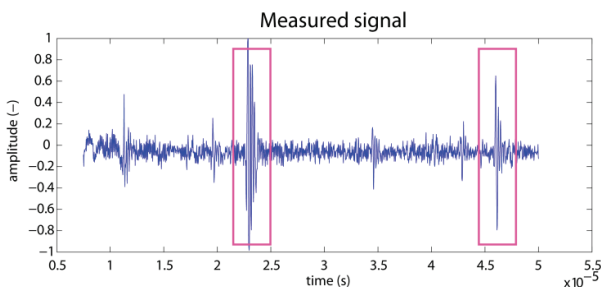


Fig. 5 Measured signal with ultrasonic backwall echoes; backwall echoes corresponding to a single and a double thickness of the specimen are presented, because of strong noise the flaw echo cannot be detected

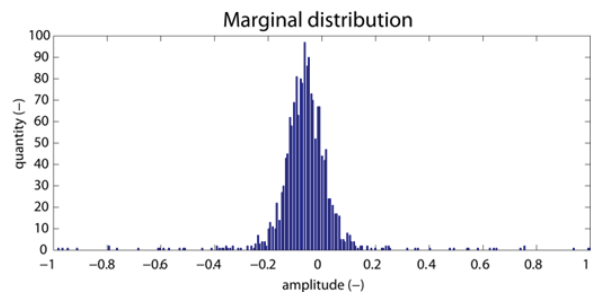


Fig. 6 Measured signal marginal distribution, the characteristics is similar to Gaussian (normal) distribution

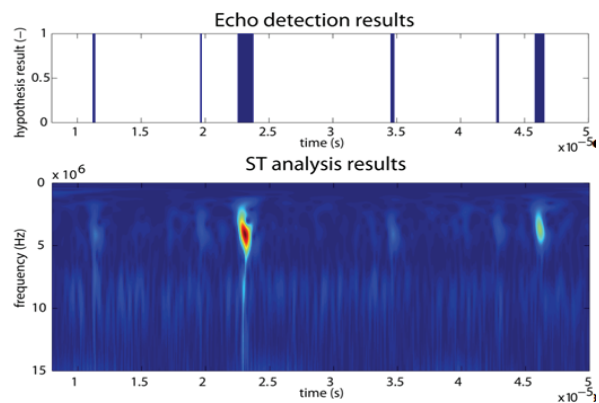


Fig. 7 Time and time-frequency analysis of the measured signal

The result of signal filtering is in Fig. 8. Backwall echoes from shear wave have yellow color. EMAT probe produced also longitudinal waves and their reflections from backwall are marked with blue color. Green color has box with echo reflected from 4 mm flat bottom hole made into the aluminum specimen, for details see Fig. 9. Backwall echoes are repeated with smaller amplitude caused by material attenuation. The reflection from the flaw simulated by the flat bottom hole is not repeated after first backwall echo. The primary condition for correct flaw echo detection is insignificance of a noise which is caused by a spurious signal reflection. If this condition is not fulfilled then detector indicates not only flaw echoes. This has to be taken as a source of a possible non-zero error rate of the proposed filtering process.

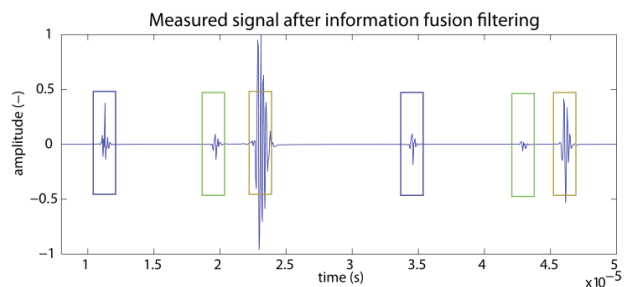


Fig. 8 Result of the applied information fusion filtering algorithm on the measured signal.

In order to compare performance of the information fusion filtering algorithm and common filtering methods signal to noise ratio is used. As a common method for comparison were used the digital filter and wavelet filtering.

Three conditions were set to design an optimal digital filter for ultrasonic signal. The first criterion was a minimal filter order to receive a short system response. The measured echo signal is limited with only 10 samples per one period of the EMAT signal. The signal of interest should

not be attenuated and therefore a zero gain has to be achieved in pass-band of the filter. Preliminary spectra analyses showed ultrasonic signal located between 4 and 6 MHz. This broadband was used as a third condition for the filter design. MATLAB Signal processing toolbox proposed a filter Chebyshev II with an order equal to 10. After correction of the phase error the filter order will be 20. The echo has at least 3 periods what correspond to 30 samples and therefore the signal is longer than response of the filter.

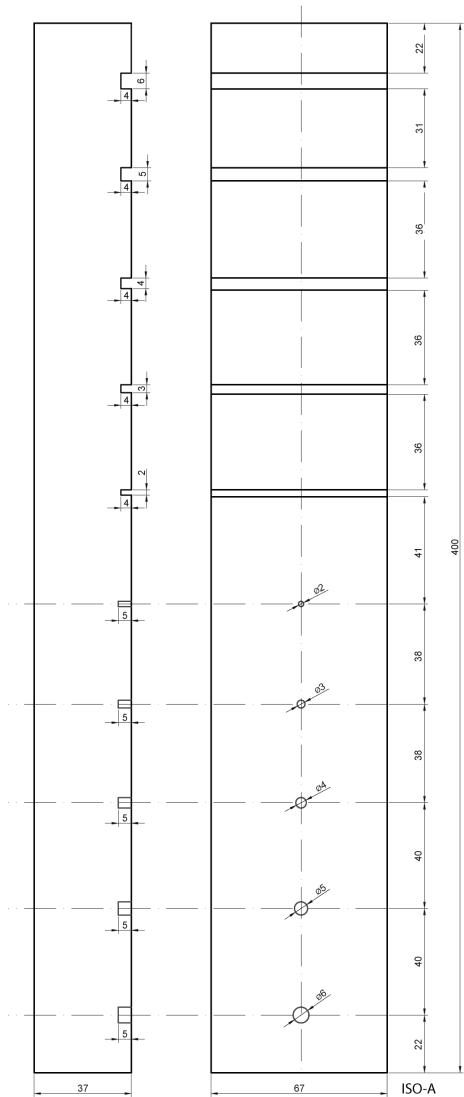


Fig. 9 Specimen made of aluminum material EN 2024 T4 standard 94,7% Al; 3.8% Cu; 1.2% Si

For wavelet filtering a discrete wavelet transformation is used with soft thresholding and SURE level estimation according to Donoho [23]. Recently published study by Matz [24] presents a further overview on wavelet filtering.

The full comparison chart is also attached in this article. As the evaluation parameter was chosen Signal to Noise Ratio (SNR) defined as

$$(14) \quad SNR = 20 \log \left( \frac{\max(x^2(i_1))}{\max(x^2(i_2))} \right)$$

where measured signal  $x$  is sampled in windows  $i_1 \in \langle o, p \rangle$  for a flaw echo and a time window  $i_2 \in \langle k, l \rangle$  for a noise area. The centers of those time windows were set to known position of the flaw echo and an expected area without the flaw echo.

The additional results are presented in Fig. 10. Each input SNR corresponds to different measured flaw. The overall results of the proposed information fusion algorithm outperformed the compared methods for input signal with low SNR.

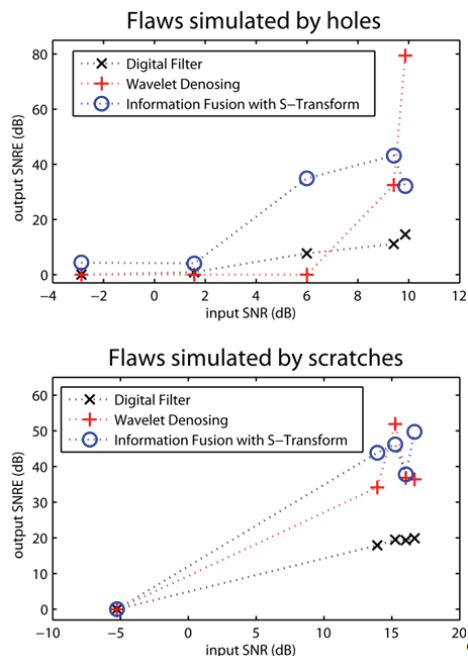


Fig. 10 Comparison chart of the proposed information fusion filtering algorithm, digital filter and wavelet denoising. Different signal input SNR corresponds to variable dimensions of presented flaws. Results are made as an average of 100 measurements for each flaw

### Conclusion

The idea of finding the best suited transform for ultrasonic signals resulted in evaluation of two main approaches with different transformation bases. Wavelet transform exhibits a progressive resolution for time-frequency analysis and its application in ultrasonic signal filtering is widely published. Recently developed S-Transform in compare to Fourier Transform improves the time-frequency resolution and therefore its theoretical properties place it between Fourier Transform and the wavelet transform. The direct implementation and application on ultrasonic signals from Electromagnetic Acoustic Transducer (EMAT) represents clue, for the study of alternative signal processing to the wavelet transform. This leads to an improved algorithm for separation of noise and the ultrasonic echo compared to commonly used filters. Within this study were introduced all principal aspects of the ultrasonic and EMAT signals. The new alternative information fusion algorithm which draws ultrasonic echoes from the noisy signal was derived. The algorithm results with a non-zero error rate occur when the amplitude of flaw echoes are smaller than the noise level. The ultrasonic measurement classification is therefore corrupted by incorrect ultrasonic findings. The proposed method achieves higher signal to noise ratio enhancement for low input SNR than using of digital filters, wavelet denoising or other discussed methods.

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## REFERENCES

- [1] S. Starman: *Utility design of EMAT probe, n. 20278*, Industrial Property office of Czech Republic, 2009.
- [2] Masahiko Hirao and Hirotugu Ogi: *EMATS for Science and Industry: Noncontacting Ultrasonic Measurements*, Kluwer Academic Publishers Group, Netherlands, 2003.
- [3] K.F. Graff: *Wave Motion in Elastic Solids*, Dover Publications, pp. 213-257, 1991
- [4] N. M. Bilgutay, U. Bencharit, J. Saniie: *Enhancement ultrasonic imaging with split-spectrum processing and polarity thresholding*, IEEE Transactions on Acoustics, Speech and Signal Processing, vol. 37, no. 10, pp. 1590-1592, 1989.
- [5] A. Hoess and H. Ermer: *Adaptive Wiener filtering for B-mode image improvement*, IEEE Ultrasonics Symposium Proceedings, vol. 2, pp. 1219-1222, 1992.
- [6] I. Daubechies: *Orthonormal Bases of Compactly Supported Wavelets*, Communications on Pure and Applied Mathematics, vo. 41, no. 7, pp. 909-996, 1988.
- [7] D.L. Donoho and I.M. Johnstone: *Adapting to Unknown Smoothness via Wavelet shrinkage*, Journal of the American Statistical Association, vol. 90, no. 432, pp. 1200-1224, 1995.
- [8] A. Abbate and J. Koay: *Signal detection and noise suppression using a wavelet transform signal processor: application to ultrasonic flaw detection*, IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control, vol. 44, no. 1, pp. 14-26, 1997.
- [9] F.J. Harris: *On the use of windows for harmonic analysis with the discrete Fourier Transform*, Proc. IEEE, vol. 66, no. 1, pp. 51-83, 1978.
- [10] R. Stockwell: *A basis for efficient representation of the S-transform*, Digital Signal processing, vol. 17, no. 1, pp. 371-393, 2007.
- [11] Soo-Chang Pei and Pai-Wei Wang: *Novel Inverse S Transform With Equalization Filter*, IEEE Transactions on Signal Processing, vol. 57, no. 10, pp. 3858-3868, 2009.
- [12] Robert A. Brown, M. Louis Lauzon and Richard Frayne: *A General Description of Linear Time-Frequency Transforms and Formulation of a Fast, Invertible Transform That Samples the Continuous S-Transform Spectrum Nonredundantly*, IEEE Transactions on Signal Processing, vol. 58, no. 1, pp. 281-290, 2010.
- [13] Song Shou-Peng and Que Pei-Wen: *Wavelet Based Noise Suppression Technique and Its Application to Ultrasonic Flaw Detection*, Ultrasonics, vol. 4, pp. 188-193, 2006.
- [14] S. M. Kay: *Fundamentals of statistical signal processing: estimation theory*, Prentice-Hall, 625 pp., 1993
- [15] H. Vincent Poor: *An Introduction to Signal Detection and Estimation*, Springer-Verlag New York, 1994.
- [16] M. Kubinyi: *Sensitivity Increasing of an Ultrasonic EMAT Materiology*, AIAA-Pegasus Conference Proceedings, Naples, 2007.
- [17] M. Schimmel and J. Gallart: *The Inverse S-Transform in Filters With Time-Frequency Localization*, IEEE Transactions on Signal Processing, vol. 53, no. 11, pp. 4417-4422, 2005.
- [18] Soo-Chang Pei and Pai-Wei Wang: *Novel inverse S-Transform with equalization filter*, IEEE Transactions on Signal Processing, vol. 57, no. 10, pp. 3858-3868, 2009.
- [19] M. Vetterli and C. Herley: *Wavelets and Filter Banks: Theory and Design*, IEEE Transactions on Signal Processing, vol. 40, no. 9, pp. 2207-2232, 1992.
- [20] R. Stockwell, L. Mansinha and R. Lowe: *Localization of the complex spectrum: the S-transform*, IEEE Transactions on Signal Processing, vol. 44, no. 4, pp. 998 -1001, 1996.
- [21] M. Schimmel and J. Gallart: *The inverse s-transform in filters with time-frequency localization*, IEEE Transactions on Signal Processing, vol. 53, no. 11, pp. 4417 - 4422, 2005.
- [22] S.-C. Pei and P.-W.Wang: *Novel inverse S-transform with equalization filter*, IEEE Transactions on Signal Processing, vol. 57, no. 10, pp. 3858 -3868, 2009.
- [23] David L. Donoho: *Adapting to unknown smoothness via wavelet shrinkage*, Tech. rep., Stanford University, 1994
- [24] V. Matz, R. Smid, S. Starman, M. Kreidl: *Signal-to-noise ratio enhancement based on wavelet filtering in ultrasonic testing*, Ultrasonics, vol. 49, no. 8, pp. 752 - 759, 2009.

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