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Structure optimization of power system using Simulated Annealing Algorithm

Abstract. *The simulated annealing algorithm is used as an optimization technique to solve the problem of total investment cost optimization, subject to the reliability constraints. The problem of optimization of the structure of a power system where redundant elements are included in order to provide a desired level of reliability is known as Redundancy Optimization Problem. The problem is presented by multi-state series-parallel systems. System reliability is defined as the ability to satisfy consumer demand which is represented as a piecewise cumulative load curve. We supposed variation of the load cumulative demand curve null. A universal generating function technique is applied to evaluate system availability.*

Abstract. *Algorytm symulowanego wyżarzania jest używany jako technika optymalizacji do rozwiązywania problemów optymalizacji kosztów przy wymuszonej niezawodności. Problem optymalizacji struktury systemu mocy z nadmiarowym elementem zabezpieczającym pożądaną niezawodność jest znany jako Redendancy Optimization Problem. Problem jest analizowany w systemie szeregowo-równoległym. Niezawodność systemu jest zdefiniowana jako zdolność spełnienia żądań konsumentów które są reprezentowane jako część skumulowanej krzywej obciążeń. Założono zerową wariację krzywej obciążeń. Zaproponowano uniwersalną technikę możliwą do zastosowania w celu oceny zdolności systemu. (**Optymalizacja struktury systemu mocy przy wykorzystaniu algorytmu symulowanego wyżarzania**)*

Keywords: Simulated Annealing Algorithm, Meta heuristic method, Redundancy optimisation, Universal generating function.
Słowa kluczowe: optymalizacji sieci mocy, symulowane wyżarzanie.

1. Introduction:

The algorithm presented in this paper solves the system structure optimization problem. To solve this combinatorial optimization problem, a Simulated Annealing algorithm is used. The algorithm provides a required level of system reliability by including redundant elements in such a way that minimized the total cost of the system.
The problem of total investment cost optimization, subject to the reliability constraints, is well known as the Redundancy Optimization Problem (ROP).

The figure 1 shows the typical series-parallel structure. However, the capacity of the system is defined by multiple, heterogeneous elements. In this situation the system can have a several range level of performance depending from perfect working to total failure; in this case it is considered as a multi-sate system (MSS).[1][2]

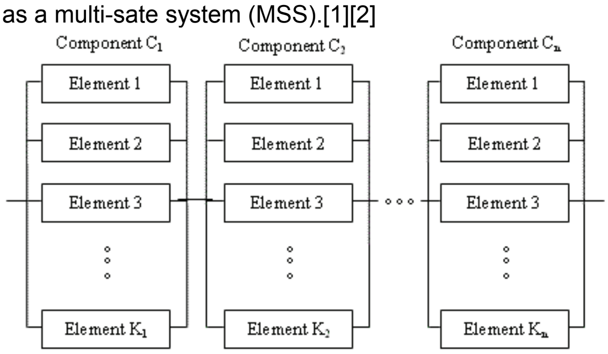

Fig. 1. Series-parallel system structure

The MSS consisting of *n* components C_i ($i = 1, 2, \dots, n$) in series arrangement. Each component C_i can contain several elements of type *i* connected in parallel from various versions which are proposed by the suppliers on the market. Each version in turn can contain identical elements put in parallel. Elements are characterised by their cost, availability and performance according to their version. Different versions of elements may be chosen for any given system component.

Thus, our study is not limited to the case where only the homogeneous elements are used. Such a limitation can be unacceptable for two reasons. First, allowing heterogeneous elements (i.e., allowed different versions to be allocated in parallel), one can obtain solution that provide the desired availability level with lower cost than in the solution with homogeneous elements. Second, in practice the designer often has to include additional elements in the existing system. It may be necessary, for example, to modernize a power system according to a new demand levels from customers or according to new reliability requirements.

2. Estimation of power system reliability using universal generating function:

The universal moment generating function (u-transform) of a discrete variable X is define as a polynomial:

$$u(z)=\sum_{j=1}^{J} p_j z^{x_j} \tag{1}$$

Where: J: Possible values of the discrete random variable X, p_j: The probability that X is equal to x_j

If only the elements with total failures are considered, and each element *i* has nominal capacity G_i and availability A_i then

$$p(X=G_i)=A_i,\quad p(X=0)=1-A_i, \tag{2}$$

If an element has only two terms, the u-function is defined as

$$u_i(z)=(1-A_i)z^0+A_i z^{G_i} \tag{3}$$

In case where elements are connected in parallel, the u-function is

$$\pi(u_1(z),u_2(z),\dots,u_n(z))=\prod_{i=1}^{n}u_i(z) \tag{4}$$

where π is the product of polynomials representing the individual u-functions.

In case where elements are connected in series, the u-function is calculated using the σ operator which is define for a pair of elements as:

$$\sigma(u_1(z),u_2(z)) = \sigma\left(\sum_{i=1}^{n} p_i z^{x_i}, \sum_{j=1}^{k} q_j z^{y_j}\right) = \sum_{i=1}^{n}\sum_{j=1}^{k} p_i q_j z^{min\{x_i, y_j\}} \quad (5)$$

One can see that, in series case, the element with the minimal capacity becomes the bottle-neck of the system. For a system containing m elements connected in series, σ can be calculated using the rule

$$\sigma(u_1(z),u_2(z),...,u_m(z)) = \sigma(\sigma(u_1(z),u_2(z)),...,u_m(z)) \quad (6)$$

The above equation evaluate the probability that the random variable X represented by polynomial u(z) defined in equation 1 exceeds a specified load demand level W

$$P(X \geq W) = \delta(u(z),W) = \sum_{x_j \geq W} p_j \quad (7)$$

To evaluate the reliability index E for the entire system, its u-function should be obtained using π and σ operators, and the δ operator should be applied for all the demand levels W_i.

3. Formulation of the power system cost optimization problem:

The multi-state system cost optimization problem (COP) under reliability constraint of electrical power system, represented in figure 1, can be formulated as follows: find the lower cost system configuration ***C***, such that the corresponding reliability less than or equal to specified (desired) value $\boldsymbol{E_0}$.

(15) **Min** $C = \sum_{i=1}^{n}\sum_{v=1}^{V_i} K_{iv} C_{iv}$

(16) **Subject to** $E \geq E_0$

where:
$i = 1, 2, …, n$: subcomponents in series.
v=1, 2, …, V_i: number of technologies available for component of type i,
C_{iv} cost of subcomponent i characterized by the technology v,
k_{iv} : numbers of parallel components (of each technology)

Where E can be calculated using equation (7).

4. Simulated Annealing Algorithm

The method of simulated annealing is an algorithm based on heuristic allowing the search for solution at a given problem. It draws its name and its inspiration from the physic of materials and more especially of the metallurgy. This process is used to improve the quality of a solid.
While seeking to reach a state of minimal energy which corresponds to a stable structure of metal, this process alternates cycles of slow cooling and reheating (annealing).

This physical idea is that a too brutal cooling can block metal in an unfavorable state (whereas a slow cooling makes it possible the molecules as well as possible to be arranged in a stable configuration). It is this same idea which is at the base of simulated annealing. To prevent that the algorithm does not remain trapped in local minima, it is made so that the temperature T = T(n) decreases slowly according to time.

In the Fifties, Metropolis et al realized the algorithm of annealing simulated to simulate the evolution of this process of physical annealing. [3]
The use for the resolution of the optimization combinatorial problems is much more recent and dates from the Eighties. It was developed by three researchers of IBM company, S. Kirkpatrick [4], C.D. Gelatt et M.P. Vecchi en 1983, and independently by V. Cerny en 1985.

4.1. Description of the algorithm:

By analogy with the physical process (thermodynamic system), the function to be minimized will become energy E of the system. One also introduces a fictive parameter, the temperature T of the system.

Leaving from a given solution, by modifying it, one obtains one second from it. Either this one improves the criterion which one seeks to optimize, one says then that one made drop the energy of the system, either this one degrades it. If one accepts a solution improving the criterion, one thus tends to seek the optimum in the neighbourhood of the starting solution. The acceptance of « a bad » solution then allows it to explore most of the space of solution and tends to avoid being locked up too quickly in the search for a local optimum.

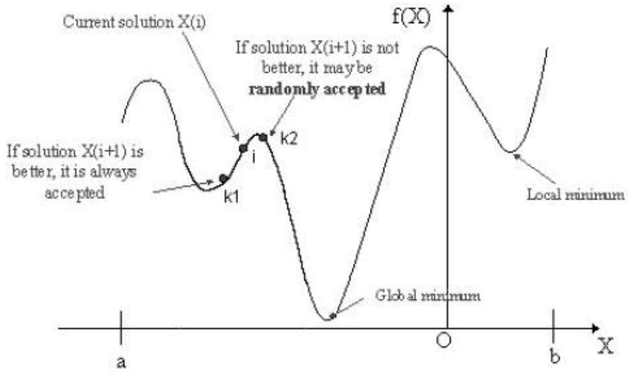

Fig. 2. Selection of a new states in simulated annealing

4.2. Flowchart of the algorithm

The figure 3 presents the flowchart of the algorithm

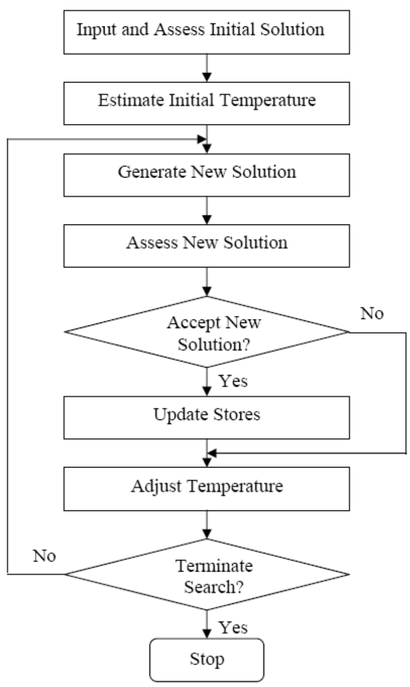

Fig. 3. Flowchart of simulated annealing

5. Application

The algorithm of simulated annealing applied to solve the power system structure is given by the figure 4.

Initialization:
Provide an initial solution (structure) S (initial configuration of Electrical Power System)
Provide an initial Temperature
Sopt = S
Calculation of the Current_Cost using Fopt = F(Sopt) (F(): objective function)
Do
  Do
    New_Structure
    Calculation of the new_Cost
    IF Δ(Current_cost - New_Cost) $\leq$ 0 THEN
    Current_Structure= New_Structure
    ELSE
    IF EXP(Current_cost - New_Cost / Temperature)> Random (0,1)
    THEN *// Accept*
    Current_Structure = New_Structure
    ELSE *// Reject*
    While repetition < Max Repetition T constant
Decrease the temperature
While Temperature < Final Temperature

Fig. 4. Simulated annealing algorithm

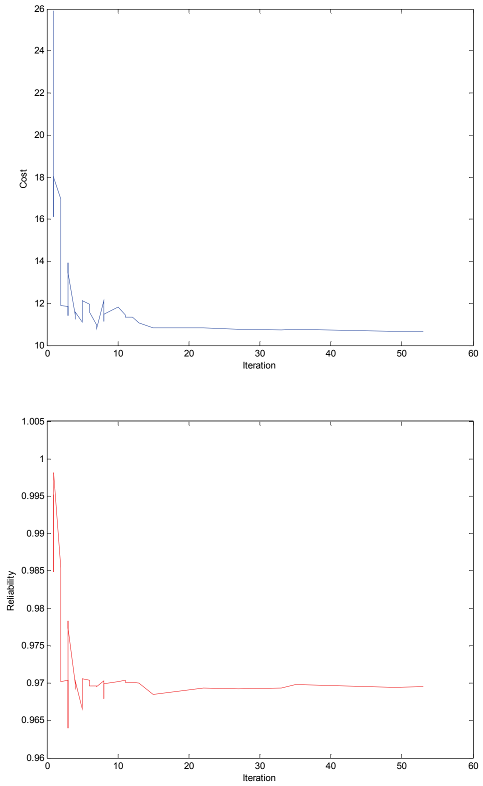

Fig. 5. Evolution of power system cost and reliability using the SA (E=0.9773)

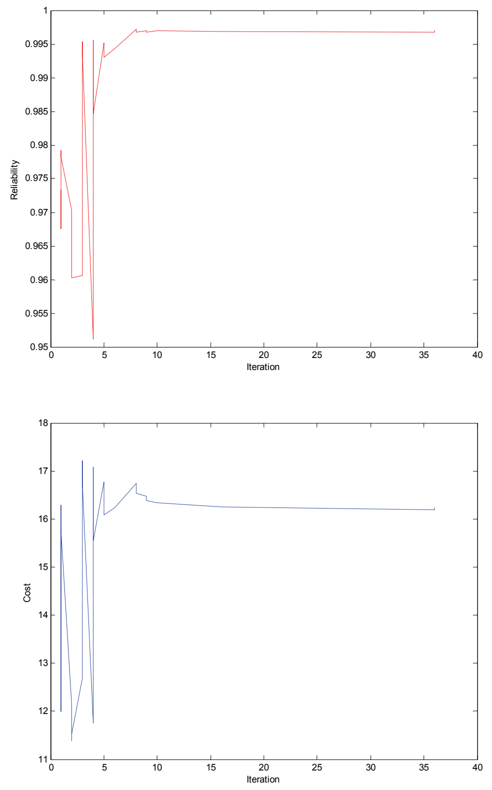

Fig. 6. Evolution of power system cost and reliability using the SA (E=0.9970)

Table 2. Parameters of the cumulative demand curve [5]

Load level (%)	100	80	50	20
Operating Time (h)	4203	788	1228	253 6
Operating Time (%)	47.9	08.9	14.0	28.9

Table 3. Parameters of the optimal solutions obtained by SA Algorithm for different desired reliability levels

E_0	E	C	Structure
0.976	0.9773	13.4440	1:3(1), 6(1), 5(1), 7(1) 2:4(2), 2(1), 3(1) 3:4(1), 1(1) 4:2(1), 5(1), 7(1), 8(1) 5:3(2), 4(1)
0.980	0.9826	15.794	1:4(2), 6(1) 2:5(1), 2(1) 3:2(2), 3(1) 4:7(3) 5:2(1), 4(1)
0.992	0.9970	16.2290	1:4(1), 7(1), 3(1) 2:2(1), 5(1), 1(1) 3:2(2), 3(1) 4:7(3) 5:4(2), 3(1)

Table 1. Data examples [5]

		Technology #								
Sub-System #	*Components*	*# 1*	*# 2*	*# 3*	*# 4*	*# 5*	*# 6*	*# 7*	*# 8*	*# 9*
1	*Reliability* **(%)** *Cost* **(Ml$)**	*0.980* *0.590*	*0.977* *0.535*	*0.982* *0.470*	*0.978* *0.420*	*0.983* *0.400*	*0.920* *0.180*	*0.984* *0.220*	*/* */*	*/* */*
2	*Reliability* **(%)** *Cost* **(Ml$)**	*0.995* *0.205*	*0.996* *0.189*	*0.997* *0.091*	*0.997* *0.056*	*0.998* *0.042*	*/* */*	*/* */*	*/* */*	*/* */*
3	*Reliability* **(%)** *Cost* **(Ml$)**	*0.971* *7.525*	*0.973* *4.720*	*0.971* *3.590*	*0.676* *2.420*	*/* */*	*/* */*	*/* */*	*/* */*	*/* */*
4	*Reliability* **(%)** *Cost* **(Ml$)**	*0.977* *0.180*	*0.978* *0.160*	*0.978* *0.150*	*0.983* *0.121*	*0.981* *0.102*	*0.971* *0.096*	*0.983* *0.071*	*0.982* *0.049*	*0.977* *0.044*
5	*Reliability* **(%)** *Cost* **(Ml$)**	*0.984* *0.986*	*0.983* *0.825*	*0.987* *0.490*	*0.981* *0.475*	*/* */*	*/* */*	*/* */*	*/* */*	*/* */*

In order to solve this problem using the Simulated Annealing Algorithm (SA), we used data given in tables 1. Table 2 contains the data of cumulative demand. The maximum numbers of components put in parallel are set to (7, 5, 4, 9, 4).

6. Conclusion and discussion:

In this paper, our objective is to define the minimal cost which provides the desired reliability $E \geq E_0$. To find the optimal structure of power system, we used the simulated annealing algorithm.

The table 3 illustrates the different structures found by the algorithm for a different reliability levels E_0. The former also shows the computed investment C and reliability E of the best power system. The format of system structure is : n: $v_1(r_i)$, …, $v_m(r_m)$, where n is a number denoting system component, r_i is the number of elements of version v_i belonging to this component.

Figure 5 and 6 illustrate two simulations made for two reliability levels. All simulations are obtained in 5000 cycles (iterations). We observed that the quality of the solution strongly depends on the parameters of the algorithm especially temperature parameter.

We supposed the variation of the load demand cumulative curve null.

We have combined a universal generating function with simulated annealing in the optimization procedure. This method, universal generating function, is used as a tool for estimating the reliability of complex systems whose components and their interactions have different physical nature.

The SA provided solutions as good as those obtained by other meta-heuristic methods.[6] [7]

Moreover, the IA acquired optimal solutions in significantly fewer evaluations (iterations or cycles) than most competitive algorithms, including the GA.

REFERENCES

[1] Ushakov, Levitin and Lisnianski 2002 “Multi-state system reliability: from theory to practice”. *Proc. of 3 Int. Conf. on mathematical methods in reliability, MMR 2002*, Trondheim, Norway, pp. 635-638.

[2] Levitin and Lisnianski, 2001 “A new approach to solving problems of multi-state system reliability optimization”. *Quality and Reliability Engineering International*, vol. 47, No. 2, pp. 93-104.

[3] N. Metropolis, A. Rosenbluth, M. Rosenbluth, A. Teller, E. Teller, J Chem. Phys. 21, 1087, 1953.

[4] S. Kirkpatrick, C. Gelatt, M. Vecchi, Optimization by simulated annealing. Science Journal , 220, 680, 1983.

[5] A. Rami, A. Zeblah et Y. Massim, “*Cost optimization of power system structure subject to reliability constraints using harmony search*”, PRZEGLĄD ELEKTROTECHNICZNY (Electrical Review), ISSN 0033-2097, R. 85 NR 4/2009.

[6] Levitin, Lisnianski, Ben-Haim and Elmakis, 1997 “Structure optimization of power system with different redundant elements”. *Electric Power Systems Research*, vol. 43, No. 1, pp.19-27.

[7] A. Rami, A. Zeblah, H. Hamdaoui, Y. Massim, F. Harrou, *“An efficient artificial immune algorithm for power system reliability optimisation*”, International Journal of Power and Energy Conversion 2009 - Vol. 1, No.2/3 pp. 178 – 197

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