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Orthogonality of functions describing electric power quantities in Budeanu's concept

Streszczenie. *W artykule przedstawiono wyniki szczegółowej analizy mocy w obwodach z niesinusoidalnymi przebiegami napięcia i prądu według koncepcji zaproponowanej przez Budeanu. Przypomniano, że osobą która pierwsza zaproponowała koncepcję sumowania mocy biernych poszczególnych harmonicznych był Bunet. Postulat ten został włączony do tzw. twierdzenia Boucherota. Na podstawie analizy obwodu zawierającego jedynie idealny induktor wykazano błędność założeń zarówno w koncepcji Boucherota jak i Budeanu. Założenie o addytywności mocy nie zawsze jest prawdziwe. Wykazano również, że zaproponowane przez Budeanu moce: bierna (reaktywna) i deformacji nie zawsze są ortogonalne. Bezkrytyczne twierdzenie o ortogonalności mocy jest nieuprawnione* ***(Ortogonalność funkcji opisujących moce według koncepcji Budeanu).***

Abstract. *The Budeanu's concept of circuit analyses with nonsinusoidal voltages and currents has been presented in details with Author's comments. It has been reminded that Bunet was the first who proposed summing up reactive powers of the harmonics of nonsinusoidal waveforms. This postulate was included in so called Boucherot theorem. Based on the analyse of circuit having the ideal inductor load it has been shown that Boucherot's as well as Budeanu's assumption that powers are always additive is not correct. It has been also shown that proposed by Budeanu vectors presenting the reactive power Q_B and deformation power D not always are orthogonal. Based on mathematical calculation Author pointed out that drawing general conclusion about orthogonality of these powers is not justifiable.*

Słowa kluczowe: koncepcja Budeanu, moc bierna, moc deformacji, moc nieaktywna, teoria mocy, ortogonalność funkcji

Keywords: Budeanu's theory, functions orthogonality, Budeanu's reactive and deformation power definitions

Introduction

It could seem that Czarnecki's publication from 1987 [3] explicitly pointed out the uselessness of Budeanu's definition [1] of reactive power in circuits with nonsinusoidal waveforms of voltage and current. In Czarnecki's opinion the reactive power in this definition does not describe a phenomenon of energy oscillation and Budeanu's deformation power does not describe deformation of current with regard to voltage. On the basis of the value of the reactive power Q_B calculated according to Budeanu, it is not possible to carry out its compensation with the help of a passive elements as in the circuits with sinusoidal waveforms of voltage and current. The successive works of Czarnecki [4,5] emphasized many ambiguities in the terms proposed by Budeanu and suggested this concept should not be used by electrical engineers in their practice.

On the other hand, the postulates of Czarnecki have not won universal recognition. The IEEE standard [7] issued in 2000 recommends the use of Budeanu's concept in the analyses of circuits with nonsinusoidal waveforms of voltage and current. However 2010 version of the Standard [8] does not support it and its Annex B holds this concept as wrong.

Therefore, a number of questions arise: was Czarnecki wrong in his proofs? Is the definition of power proposed by Budeanu right? And does it describe real physical phenomena occurring in circuits?

The Author of the present paper knows neither scientific publications supporting Czarnecki's arguments nor polemical scientific publications against them . The postulate of giving up Budeanu's concept neither aroused scientific support nor met an explicit objection. Why? In the Author's opinion the main difficulty lies in the lack of an access to Budeanu' publication from 1927, despite the fact that numerous articles describing power states in electrical circuits frequently cite this publication. After a long search, the Author managed to get the copy of Budeanu's book [1], which is in the British Library in London, and on its basis presents his own observations.

Budeanu's proposal of the description of power quantities in circuits with nonsinusoidal waveforms of voltage and current

In his book [1], Budeanu uses the names and symbols well-known or introduced by himself in the twenties of the previous century. The square equation of power states in his notation has the following form

(1) $$P_f = \sqrt{{P_a}^2 - P^2}$$

where: P_f - is fictitious power, P_a - apparent power and P -is active power.

In order to make the text more understandable, the currently used symbols and names will be used in the description of Budeanu's concept. Thus, the equation (2) is contemporarily written as

(2) $$N = \sqrt{S^2 - P^2}$$

where: N stands for non-active power, S stands for apparent power and P stands for active/real power

Budeanu's concept of the description of power states in circuits with nonsinusoidal waveforms contains a number of original recommendations. Budeanu provided the definitions of:

- similar functions (Fr. fonctions semblables): two periodical nonsinusoidal functions $u(t)$ and $i(t)$ are similar, when in their decompositions into a Fourier's series, the harmonics of the same order n meet two conditions:

(3) $$\frac{U_n}{I_n} = const$$

(4) $$\phi_n = \phi_{un} - \phi_{in}$$

Budeanu's "similar functions" are illustrated by Fig. 1,

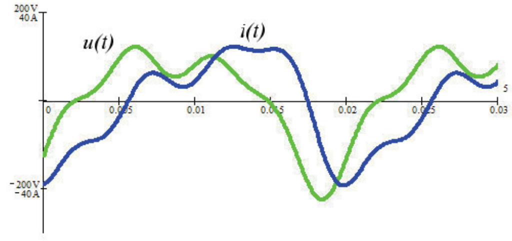

Fig. 1 Illustration of Budeanu's similar (but not in phase) functions

where: $I_n = 0,2\ U_n$, $\phi_{in} = \phi_{un} - \frac{\pi}{3}$

$$u(t) = \sqrt{2}\ 90 \sin(\omega\, t + \frac{\pi}{3}) + \sqrt{2}\ 40 \sin(2\ \omega\, t + \frac{\pi}{5}) +$$

$$\sqrt{2}\ 10 \sin(3\ \omega\, t - \frac{\pi}{6}) + \sqrt{2}\ 20 \sin(4\ \omega\, t - \frac{\pi}{8})\ \mathrm{V}$$

$$i(t) = \sqrt{2}\ 18 \sin(\omega\ t) + \sqrt{2}\ 8 \sin(2\ \omega\ t - \frac{\pi}{15}) +$$

$$\sqrt{2}\ 2 \sin(3\ \omega\ t - \frac{\pi}{2}) + \sqrt{2}\ 4 \sin(4\ \omega\ t - \frac{11}{24}\pi)\ \mathrm{A}$$

- functions in "phase" (Fr. fonctions en phase): two particular periodical nonsinusoidal functions *u*(*t*) and *i*(*t*) are in "phase" (Fr. en phase), when in their decompositions into a Fourier's series the harmonics of the same order n have the same initial phase $\phi_{un} = \phi_{in}$ for instance:

(5)
$$u(t) = \sqrt{2}U_1 \sin(\omega t - \phi_1) + \ldots$$
$$\ldots + \sqrt{2}U_m \sin(m\omega t - \phi_m) + \sqrt{2}U_n \sin(n\omega t - \phi_n)$$

(6)
$$i(t) = \sqrt{2}I_1 \sin(\omega t - \phi_1) + \ldots$$
$$\ldots + \sqrt{2}I_m \sin(m\omega t - \phi_m) + \sqrt{2}I_n \sin(n\omega t - \phi_n)$$

For the function in "phase", the square equation of power (2) has the following form:

(7)
$$N^2 = S^2 - P^2 = \sum U_n{}^2 \sum I_n{}^2 - \left(\sum U_n I_n\right)^2 =$$
$$= \sum_n \sum_m \left(U_n I_m - U_m I_n\right)^2$$

where U_n, U_m, I_n, I_m are rms values of the harmonics of voltage and current of the orders n and m;

Budeanu's functions "in "phase" are illustrated by Fig. 2

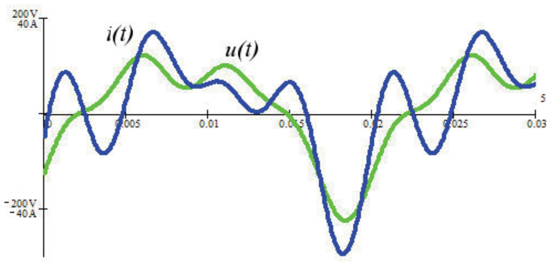

Fig. 2 Illustration of Budeanu's functions in "phase" (but not similar) where:

$$u(t) = \sqrt{2}\ 90 \sin(\omega\, t + \frac{\pi}{3}) + \sqrt{2}\ 40 \sin(2\ \omega\, t + \frac{\pi}{5}) +$$

$$\sqrt{2}\ 10 \sin(3\ \omega\, t - \frac{\pi}{6}) + \sqrt{2}\ 20 \sin(4\ \omega\, t - \frac{\pi}{8})\ \mathrm{V}$$

$$i(t) = \sqrt{2}\ 16 \sin(\omega\, t + \frac{\pi}{3}) + \sqrt{2}\ 5 \sin(2\ \omega\, t + \frac{\pi}{5}) +$$

$$\sqrt{2}\ 12 \sin(3\ \omega\, t - \frac{\pi}{6}) + +\sqrt{2}\ 10 \sin(4\ \omega\, t - \frac{\pi}{8})\ \mathrm{V}$$

$$\phi_{in} = \phi_{un}$$

- functions remaining in "quadrature" (Fr. fonctions en quadrature): two nonsinusoidal functions *u*(*t*) and *i*(*t*) remain in "quadrature" (Fr. en quadrature), when in their decompositions into Fourier' series particular harmonics of one function are shifted by an angle $\pi/2$ in relation to the harmonics of the second function corresponding to them. For example, the notation of the function in "quadrature" is:

(8) $$u(t) = \sqrt{2}U_1 \sin(\omega t - \phi_1) + \ldots + \sqrt{2}U_n \sin(n\omega t - \phi_n)$$

(9) $$i(t) = \pm\sqrt{2}I_1 \cos(\omega t - \phi_1) \pm \ldots \pm \sqrt{2}I_n \cos(n\omega t - \phi_n)$$

For the functions remaining in "quadrature" the square equation of power (2) has the following form:

(10) $$N = \sqrt{\sum U_n{}^2 \sum I_n{}^2} = S$$

Budeanu's functions remaining in "quadrature" are illustrated by Fig. 3

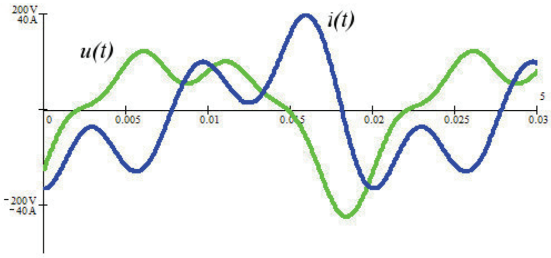

Fig. 3 Illustration of Budeanu's functions in "quadrature" (but not similar), where:

$$u(t) = \sqrt{2}\ 90 \sin(\omega\, t + \frac{\pi}{3}) + \sqrt{2}\ 40 \sin(2\ \omega\, t + \frac{\pi}{5}) +$$

$$\sqrt{2}\ 10 \sin(3\ \omega\, t - \frac{\pi}{6}) + +\sqrt{2}\ 20 \sin(4\ \omega\, t - \frac{\pi}{8})\ \mathrm{V}$$

$$i(t) = \sqrt{2}\ 16 \sin(\omega\, t - \frac{\pi}{6}) + \sqrt{2}\ 5 \sin(2\ \omega\, t - 0{,}3\ \pi) +$$

$$\sqrt{2}\ 12 \sin(3\ \omega\, t - \frac{2}{3}\pi) + \sqrt{2}\ \sin(4\ \omega\, t - \frac{5}{8}\pi)\ \mathrm{A}$$

$$\phi_{in} = \phi_{un} - \frac{\pi}{2}$$

- reactive power and deformation power - analyzing an electrical circuit of constant elements R, L, C (time - invariant load) of the terminal voltage *u*(*t*) and the current *i*(*t*) of the form:

(11)
$$u(t) = \sqrt{2}U_1 \sin(\omega t - \phi_1) + \ldots$$
$$\ldots + \sqrt{2}U_m \sin(m\omega t - \phi_m) + \sqrt{2}U_n \sin(n\omega t - \phi_n)$$
$$i(t) = \sqrt{2}I_1 \sin(\omega t - \phi_1 - \varphi_1) + \ldots$$
$$\ldots + \sqrt{2}I_m \sin(m\omega t - \phi_m - \varphi_m) + \sqrt{2}U_n \sin(n\omega t - \phi_n - \varphi_n)$$

Budeanu obtains the following equation describing the apparent power S:

(12)
$$S^2 = \left(U_1{}^2 + \ldots + U_m{}^2 + U_n{}^2\right) \cdot \left(I_1{}^2 + \ldots + I_m{}^2 + I_n{}^2\right) =$$
$$= \left(\sum_n U_n I_n \cos\varphi_n\right)^2 + \left(\sum_n U_n I_n \sin\varphi_n\right)^2 +$$
$$+ \sum_n \sum_m \left[U_m{}^2 I_n{}^2 + U_n{}^2 I_m{}^2 - 2U_m U_n I_m I_n \cos(\varphi_m - \varphi_n)\right]$$

Budeanu names particular components of the equation (12):

- real (active) power

(13) $$\sum_n U_n I_n \cos\varphi_n = P$$

- reactive power

(14) $$\sum_n U_n I_n \sin\varphi_n = Q_B$$

- deformation power

(15) $$\sqrt{\sum_n \sum_m [U_m^2 I_n^2 + U_n^2 I_m^2 - 2U_m U_n I_m I_n \cos(\varphi_m - \varphi_n)]} = D$$

and emphasizes the fact that the concept of summing up reactive powers of particular harmonics of the decomposition of the nonsinusoidal waveform into a Fourier's series is in accordance with Boucherot theorem.

Furthermore, Budeanu repeatedly states that in his opinion "reactive power is connected with the energy transfer between the circuit and the external medium formed by electrostatic and electromagnetic fields; this energy has the finite mean value" and in another place that it refers to the "energy of changeable instantaneous values, yet always of the same kind and having the finite mean value and which represent the intrinsic energy (Fr. l'énergie intrinsèque) occurring in the form of electrostatic or electromagnetic form".

Comments on Budeanu's concept

Budeanu's concept versus Boucherot theorem

Paul Boucherot (1869-1943) was a man who introduced such a concept of reactive power: in an AC electrical network, ***the total active power is the sum of individual active powers, the total reactive power is the sum of individual reactive powers, but the total apparent power is NOT equal to the sum of individual apparent powers*** (the statement known as the theorem of Boucherot). Thus, it was Boucherot and not Budeanu, who was the creator of the concept of summing up reactive powers of the harmonics of nonsinusoidal waveforms of voltage and current in an electrical circuit. This concept was approved of by many other scholars, which Budeanu points out in his work [1]. Budeanu explicitly writes that he is aware of inadequacy of mapping the real physical phenomena by this concept. On the other hand, he thinks that other concepts, more complex with regard to calculations, do not provide a better physical interpretation, either.

In Author's opinion Budeanu artificially transformed mathematical equations in such a way so that their form is in conformity with Boucherot theorem. An illustration to this observation can be the following operation performed by Budeanu: for the reactive load (functions of voltage and current of a given harmonic are in "quadrature") the apparent power S is equal to the non-active power N (10), which can be written down as:

(16) $$N^2 = \sum_n U_n{}^2 \sum_n I_n{}^2 = \sum_n U_n{}^2 I_n{}^2 + \sum_{n \neq m} U_m{}^2 I_n{}^2 = \\ = \sum_n Q_n{}^2 + \sum_{n \neq m} U_m{}^2 I_n{}^2$$

However, Budeanu did not stop with the form (16) and he transformed it further knowing that:

(17) $$\sum_n Q_n{}^2 = \left(\sum_n Q_n \right)^2 - 2 \sum_{n \neq m} U_m I_n U_n I_m$$

and

(18) $$\sum_{n \neq m} U_n{}^2 I_m{}^2 = \sum_{n \neq m} \left(U_n I_m - U_m I_n \right)^2 + 2 \sum_{n \neq m} U_m I_n U_n I_m$$

When we put the equations (17) and (18) into the equation (16), we obtain:

(19) $$N^2 = \sum_n Q_n{}^2 + \sum_{n \neq m} U_n{}^2 I_m{}^2 = \left(\sum_n Q_n \right)^2 - 2 \sum_{n \neq m} U_m I_n U_n I_m + \\ + \sum_{n \neq m} \left(U_n I_m - U_m I_n \right)^2 + 2 \sum_{n \neq m} U_m I_n U_n I_m$$

which is the Budeanu's formula of the form:

(20) $$N^2 = \sum_n Q_n{}^2 + \sum_{n \neq m} U_n{}^2 I_m{}^2 = \left(\sum_n Q_n \right)^2 + \sum_{n \neq m} \left(U_n I_m - U_m I_n \right)^2$$

The first of the two (squared) elements of the obtained expression for non-active power (20) is what is called by Budeanu, reactive power. It concerns the waveforms in which each current harmonics is shifted by an angle $\pi/2$ in relation to voltage harmonics (that is: remaining in quadrature), whereas the time waveforms of current and voltage are similar functions (Fr. semblable). The other element, called by Budeanu deformation power, is non-active power, where current and voltage are "in phase", but the waveforms are not similar (Fr. non semblable).

Deformation power D

On the basis of the definition of power [equations (13) (14) and (15)], Budeanu proposes the notation of the square equation of power (2) in the following form:

(21) $$S^2 = P^2 + Q_B{}^2 + D^2$$

From the notation (21), conclusions are drawn regarding orthogonality of vectors representing the powers P, Q_B and D. On the basis of this, one can consider a 3-dimensional power-space with P, Q ,D as orthogonal coordinates. The two-dimensional power-space with P, Q treated as orthogonal coordinates does not raise any doubts (similarly to the power-space with orthogonal coordinates P and N). Yet, in the literature on this subject there is no mathematical proof of the orthogonality of powers represented by vectors of the lengths Q_B and D, respectively. Budeanu did not provide such a proof in his book [1]. The consequence of the definition of deformation power (15) by Budeanu is solely the concept of the notation of a given number accepted by him: for instance using this convention Budeanu would write down the number 27 as $27 = \left(\sqrt{27} \right)^2$.

Therefore, the Author poses the question: is the vector represented by the reactive power Q_B orthogonal to the vector representing deformation power D ?, which means: is $\vec{Q}_B \perp \vec{D}$?

There are two meanings of orthogonality:

- if we consider functions e.g. $u(t)$ and $i(t)$ on the plain / space L^2, two functions are orthogonal if:

(22) $$\int_0^T u(t) i(t) dt = (u, i) = 0$$

This inner product is related to any function: $u(t)$, $i(t)$, so to $p(t)$ as well. One can carry out the decomposition as follows

(23) $$p(t) = p_a(t) + p_n(t)$$

and prove whether the functions $p_a(t)$ and $p_n(t)$ are orthogonal or not. Similarly, if the decomposition is as follows

(24) $$p(t) = p_a(t) + q_B(t) + d(t)$$

one can deliver the proof that:

(25) $$\int_0^T p_a(t) q_B(t) dt = (p_a, q_B) = 0$$

(26) $$\int_0^T q_B(t) d(t) dt = (q_B, d) = 0$$

- if we consider two vectors on the plain / space L^2 having the length e.g. U and I, I_a and I_q, P and Q_B or Q_B and D. Two vectors in the space are orthogonal if their inner product e.g. $< \vec{Q}_B, \vec{D} >$ is zero. This situation is denoted as $\vec{Q}_B \perp \vec{D}$.

Let us consider the following circuit (Fig. 4) in which:

$$u(t)=u_1(t)+u_3(t)=15\sqrt{2}\sin\left(\omega t-\frac{\pi}{4}\right)+20\sqrt{2}\sin\left(3\omega t-\frac{\pi}{5}\right)$$

and time-invariant load has the inductive character L=4(mH)

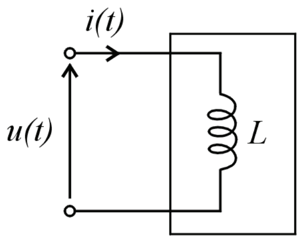

Fig. 4 The circuit with nonsinusoidal voltage and time–invariant inductive load

For given voltage *u*(*t*) , the current in the circuit from Fig. 4 has the following form:

$$i(t)=i_1(t)+i_3(t)=11.937\sqrt{2}\sin\left(\omega t-\frac{\pi}{4}-\frac{\pi}{2}\right)+$$
$$+5.30\sqrt{2}\sin\left(3\omega t-\frac{\pi}{5}-\frac{\pi}{2}\right)$$

The instantaneous power *p*(*t*) is equal:

(27) $$p(t)=u(t)i(t)=[u_1(t)+u_3(t)][i_1(t)+i_3(t)]= \\ =p_{11}(t)+p_{13}(t)+p_{31}(t)+p_{33}(t)$$

where:

$p_{11}(t)=u_1(t)i_1(t),\ p_{13}(t)=u_1(t)i_3(t),$
$p_{31}(t)=u_3(t)i_1(t),\ p_{33}(t)=u_3(t)i_3(t).$

The waveforms of voltage, current and the instantaneous power in the circuit from Fig. 4 are illustrated by Fig. 5.

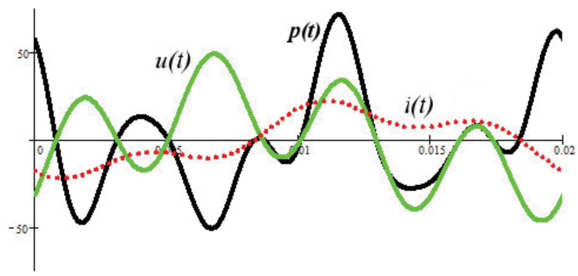

Fig. 5 Voltage u(t), current i(t) and instantaneous power p(t) in the circuit from Fig. 4

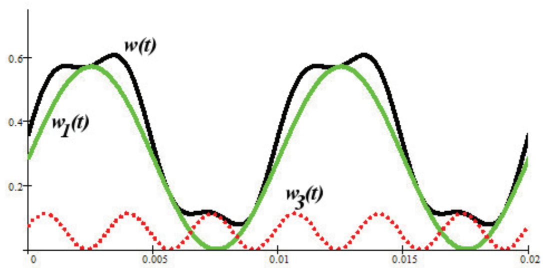

Fig. 6 Time waveforms of energy transfer w(t) and its components w1(t), w3(t) in the circuit from Fig. 4

The energy transferred from the source to the load is the energy of the magnetic field. Let assume that the instantaneous value of magnetic energy is additive and equal

(28) $$w(t)=w_1(t)+w_2(t)=0,5Li_1^2+0,5Li_2^2$$

The time waveforms of energy transfer in the circuit from Fig. 4 are illustrated by Fig. 6.

Denoting the transfer speed of energy of the magnetic field *w*(*t*) in the circuit by $p_m(t)$, that is: $p_m(t)=dw(t)/dt$, it is possible to ask a question: what components of the power $p(t)$ transfer the energy of the magnetic field in the circuit from Fig. 4? Fig. 7 confirms Budeanu's concept that the speed of changes of the magnetic field $p_m(t)$ is equal to the sum of powers of instantaneous harmonics: in the circuit from Fig. 4 these will be the first harmonic and the third harmonic, that is $p_{11}(t)+p_{33}(t)$ (Fig. 7). This is a confirmation that Boucherot's and Budeanu's concepts are based on assumption that powers are always additive.

According to physics the equation (28) is wrong and instantaneous value of magnetic energy is equal

(29) $$w(t)=0,5Li^2=0,5L(i_1+i_2)^2=0,5Li_1^2+Li_1i_2+0,5Li_2^2$$

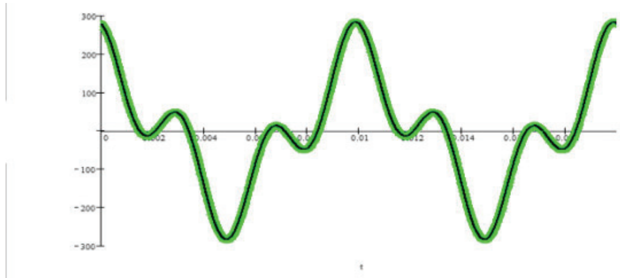

Fig.7 Time waveforms of powers $p_m(t)$ and $[p_{11}(t)+p_{33}(t)]$ in the circuit from Fig. 4

However, Budeanu's concept, in which the instantaneous reactive power $q_B(t)$ corresponds to the summing up of instantaneous powers of particular harmonics, enables the notation of the instantaneous power $p(t)$ in the form

(30) $$p(t)=u(t)i(t)=[u_1(t)+u_3(t)][i_1(t)+i_3(t)]= \\ =p_{11}(t)+p_{33}(t)+p_{31}(t)+p_{13}(t)=q_B(t)+d(t)$$

where:

$p_{11}(t)+p_{33}(t)=q_B(t)$
$p_{31}(t)+p_{13}(t)=d(t)$

An equivalent of the instantaneous power q_B(t) is vector $\vec{Q}_B$ of the length Q_B, whereas of the instantaneous power *d*(*t*) – vector $\vec{D}$ of the length D. Both the instantaneous powers q_B (*t*) and d(*t*) have the mean value for the period T equal to zero that is, $\int_0^T q_B(t)=0$, $\int_0^T d(t)=0$, whereas these powers are not orthogonal, for the data in the example:

$$\int_0^T q_B(t)d(t)=-44.578$$

The lack of orthogonality of vectors $\vec{Q}_B$ and $\vec{D}$ of the lengths Q_B and D was noticed indirectly by Czarnecki [3] when he wrote: "...*any reactive compensator changes the reactive power and deformation power at the same time*". If any reactive compensator affects both reactive and deformation powers, it means that it affects the non-active power N and not only the reactive power Q_B as it would be if the vectors of the reactive power $\vec{Q}_B$ and deformation power $\vec{D}$ were orthogonal. The calculation of power according to Budeanu's concept for various waveforms of nonsinusoidal voltage $u(t)$ in the circuit from Fig. 4 shows an interesting regularity: the lack of orthogonality of the instantaneous powers q_B (t) and $d(t)$ does not always appear.

Compensation of the reactive power Q_B, elimination of the deformation power D

A consequence of the lack of orthogonality of the reactive power $q_B(t)$ and the deformation power $d(t)$ only in special cases we have [6]:

- the reactive power is not equal to zero ($Q_B \neq 0$), but the waveforms are formatted, then (D=0) so $S^2 = P^2 + Q_B^2$,
- the reactive power is equal to zero (Q_B=0), but the waveforms are deformatted, then ($D \neq 0$), and $S^2 = P^2 + D^2$,
- the reactive power is equal to zero (Q_B=0) and moreover, the waveforms are formatted, then (D=0) so $S = P$.

Traditionally, passive shunt compensator (e.g. capacitors) can only be used to reduce the non–active power *N*.

Conclusions

It has been shown that Boucherot's as well as Budeanu's concept were based on incorrect assumption that instantaneous powers are always additive. According to that assumption each harmonic generates some magnetic field what is not in accordance with physical phenomena. In fact, powers not always meet superposition property in circuits with nonsinusoidal voltage and current waveforms. Additionally, it has been proofed based on analysis of Budeanu's concept of the description of power quantities in circuit with nonsinusoidal waveforms of voltage and currents, that only in special cases the waveforms $q_B(t)$ and $d(t)$ and at the same time vectors $\vec{Q}_B$ and $\vec{D}$ are orthogonal. Due to this fact, drawing a general conclusion from the equation (21) about the orthogonality of these powers is not justifiable.

It is worth pointing out that issued in March, 2010 a new IEEE Standard [8] does not mention Budeanu's concept

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