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Fryze’s compensator and Fortescue transformation

***Abstract**. The paper offers an algorithm for calculating the parameters of $\triangle-$ optimal Fryze’s compensator with LC elements under asymmetrical voltage and unbalanced load. A symmetrical basis was proposed which allows to apply the Fortescue transformation method in a unitary form for the three-wire unbalanced system. Results of numeric modelling are given.*

***Streszczenie.** W artykule przedstawiono algorytm obliczania $\triangle-$ optymalnego kompensatora Fryzego z elementami LC przy niesymetrycznym napięciu i obciążeniu. Zaproponowano symetryczną podstawę, która umożliwia zastosowanie transformacji Fortescue w jednolitym kształcie dla trójprzewodowego układu niesymetrycznego. Przedstawiono również wyniki modelowania numerycznego. (**Kompensator Fryzego a transformacja Fortescue**)*

Keywords: active and inactive current; asymmetrical voltage and load; optimal compensator; symmetrical components.
Słowa kluczowe: prąd czynni i nieaktywny, asymetryczne napięcie i obciążenie, kompensator optymalny, składowe symetryczne.

Introduction

Over the past decades the number of electricity consumers with imperfect load (reactive, unbalanced, nonlinear, non-stationary) increased significantly, which, in turn, led to the quality deterioration of the energy delivered in three-phase distribution networks. Connection of asymmetrical loads without compensating devices results in line current asymmetry, unbalanced voltage, additional losses and pulsation of instantaneous power (IP). Symmetrization, i.e. the compensation of loads designed for symmetrical voltage, does not provide the unit power factor in the case of asymmetrical voltage. It is known that Steinmetz symmetrization scheme for a single-phase load [1-4] provides the unit power factor for symmetrical voltage (the modified apparent power equals the active one). If voltage is not symmetric, then this is not the case [2-3]

Fryze's approach [5] allows to get a unit power factor under any voltage. The approach became the key one in the Currents’ Physical Components (CPC) Power Theory [6-9]. Within the framework of CPC Power Theory the parameters of a compensator with LC element alone are determined in [7, 8] under symmetrical sinusoidal voltage for any triangle type load. It is argued erroneously in [10] that it is impossible to provide such a compensator within the framework of CPC Power Theory if voltage were asymmetric.

This paper offers $\triangle-$ Fryze’s compensator with LC elements alone under asymmetrical sinusoidal voltage. The suggested algorithm was worked out with the use of the method of Fortescue transformation [11,12]. A two-dimensional basis of symmetrical components was introduced in order to describe the two-dimensional energy processes in the three-wire system unbalanced. This allowed to determine the transformation (Fortescue type) for three-wire system and to show how a compensator with reactive elements alone can be provided for any asymmetrical sinusoidal voltage.

Fryze’s decomposition under Asymmetrical Sinusoidal Voltage

Fryze suggested to represent current in a circuit as a sum of two components: an active one and a reactive one (or inactive, to be more precise) [5]. Originally Fryze studied a decomposition of this kind for a single-phase circuit. Fryze’s approach can be naturally generalized to polyphase circuits with $T-$ periodic waveforms [6].

Let us consider the modern interpretation of the approach for three-phase systems with sinusoidal waveforms of current and voltage [13-17]

(1) $$\boldsymbol{i}(t)=(i_a(t),i_b(t),i_c(t))^{\top}=\sqrt{2}\Re e[\boldsymbol{I}e^{j\omega t}],$$

(2) $$\boldsymbol{u}(t)=(u_a(t),u_b(t),u_c(t))^{\top}=\sqrt{2}\Re e[\boldsymbol{U}e^{j\omega t}].$$

The sinusoidal waveforms of voltage and current (1-2) in a three-wire cross-section $<a,b,c>$ are completely defined by three-dimensional complex vectors (the current and voltage 3D-phasors)

(3) $$\boldsymbol{U}=\begin{bmatrix}\dot U_a\\ \dot U_b\\ \dot U_c\end{bmatrix}=\begin{bmatrix}U_a e^{j\psi_a}\\ U_b e^{j\psi_b}\\ U_c e^{j\psi_c}\end{bmatrix},\qquad \boldsymbol{I}=\begin{bmatrix}\dot I_a\\ \dot I_b\\ \dot I_c\end{bmatrix}=\begin{bmatrix}I_a e^{j\varphi_a}\\ I_b e^{j\varphi_b}\\ I_c e^{j\varphi_c}\end{bmatrix}.$$

3D - phasors (3) are defined as

(4) $$\boldsymbol{U}=\frac{\sqrt{2}}{T}\int_0^T \boldsymbol{u}(t)e^{-j\omega t}dt,\quad \boldsymbol{I}=\frac{\sqrt{2}}{T}\int_0^T \boldsymbol{i}(t)e^{-j\omega t}dt.$$

Here and further $\top$ is the transposition symbol, T denotes the period ($T\omega=2\pi$).The rms values of (1-2) are equal to the current and voltage 3D- phasor norms

(5) $$I=|\boldsymbol{I}|=\sqrt{|\dot I_a|^2+|\dot I_b|^2+|\dot I_c|^2},$$

(6) $$U=|\boldsymbol{U}|=\sqrt{|\dot U_a|^2+|\dot U_b|^2+|\dot U_c|^2}.$$

In sinusoidal operating mode (under sinusoidal conditions) the instantaneous power (IP)

(7.a) $$p(t)=u_a(t)i_a(t)+u_b(t)i_b(t)+u_c(t)i_c(t)$$

is equal to

(7.b) $$p(t)=\Re e[\dot S+\dot N e^{j2\omega t}]=P+\Re e[\dot N e^{j2\omega t}].$$

Here the standard complex power (SCP) and the complex pulsation power (PP) of the total current are equal to

(8) $$\dot S=\boldsymbol{U}^{\top}\boldsymbol{I}^*=\dot U_a I_a^*+\dot U_b I_b^*+\dot U_c I_c^*\qquad \dot S=P+jQ,$$

(9) $$\dot N=\boldsymbol{U}^{\top}\boldsymbol{I}=\dot U_a\dot I_a+\dot U_b\dot I_b+\dot U_c\dot I_c.\quad \dot N=|\dot N|e^{j\arg\dot N}.$$

The average (active) power

(10) $$P = \frac{1}{T}\int_{\tau}^{\tau+T} p(t)dt = \frac{1}{T}\int_{\tau}^{\tau+T} \boldsymbol{u}(t)^{\top}\boldsymbol{i}(t)dt$$

can be calculated as the real part $P = \Re e\dot{S}$ of the complex power $\dot{S} = P + jQ$ (inner product of current and voltage 3D- phasors (3)). Here, the asterisk (*) denotes the complex conjugation operation. $\boldsymbol{I}^* = (I_a^*, I_b^*, I_c^*)^{\top}$ is the complex conjugate current 3D- phasor.

In the 3-wire cross-section $< a,b,c >$ the active Fryze's current is defined by the 3D-phasor as

(11) $$\boldsymbol{i}_a(t) = \sqrt{2}\Re e[\boldsymbol{I}_a e^{j\omega t}], \qquad \boldsymbol{I}_a = G_a\boldsymbol{U} = \frac{P}{|\boldsymbol{U}|^2}\boldsymbol{U},$$

where coefficient $G_a = P/|\boldsymbol{U}|^2$ is equal to the equivalent conductivity of the active current.

The active Fryze current (11) supplies the same average power(10) as the total current (1)

(12) $$P = \frac{1}{T}\int_{\tau}^{\tau+T} \boldsymbol{i}_a^{\top}(t)\boldsymbol{u}(t)dt = G_a|\boldsymbol{U}|^2 = |\boldsymbol{I}_a||\boldsymbol{U}|.$$

Under sinusoidal condition with asymmetrical voltage the following relation for the inactive Fryze current holds

(13) $$\boldsymbol{i}_F(t) = \sqrt{2}\Re e[\boldsymbol{I}_F e^{j\omega t}] \qquad \boldsymbol{I}_F = \frac{-jQ}{|\boldsymbol{U}|^2}\boldsymbol{U} + \frac{\boldsymbol{D}\times\boldsymbol{U}^*}{|\boldsymbol{U}|^2}.$$

The outer product of the voltage and current 3D- phasors

(14) $$\boldsymbol{D} = \boldsymbol{U}\times\boldsymbol{I} = \begin{bmatrix} \dot{U}_b\dot{I}_c - \dot{U}_c\dot{I}_b \\ \dot{U}_c\dot{I}_a - \dot{U}_a\dot{I}_c \\ \dot{U}_a\dot{I}_b - \dot{U}_b\dot{I}_a \end{bmatrix}.$$

defines the imbalance power vector; $\boldsymbol{U}^* = (U_a^*, U_b^*, U_c^*)^{\top}$ is the complex conjugated voltage 3D-phasor. We have the decomposition

(15) $$\boldsymbol{I} = \boldsymbol{I}_a + \boldsymbol{I}_F = \frac{P}{|\boldsymbol{U}|^2}\boldsymbol{U} + \frac{Qe^{-j\pi/2}}{|\boldsymbol{U}|^2}\boldsymbol{U} + \frac{\boldsymbol{D}\times\boldsymbol{U}^*}{|\boldsymbol{U}|^2}$$

The active and inactive currents are mutally orthogonal, that is why we have for the rms currents values

(16) $$|\boldsymbol{i}|^2 = |\boldsymbol{i}_a|^2 + |\boldsymbol{i}_F|^2, \qquad |\boldsymbol{I}|^2 = |\boldsymbol{I}_a|^2 + |\boldsymbol{I}_F|^2.$$

Buchholz's apparent power [18] is defined as the product of the current and voltage rms values

(17) $$S_B = I\cdot U = |\boldsymbol{i}|\cdot|\boldsymbol{u}|.$$

By the Pythagorean Theorem for rms currents (16) one arrives at the following quadratic decomposition of Buchholz's apparent power (power equation)

(18) $$S_B^2 = P^2 + Q_F^2.$$

The inactive power (inactive current) by Fryze

(19) $$Q_F = I_F\cdot U = |\boldsymbol{I}_F|\cdot|\boldsymbol{U}|$$

defines additional losses for 1 Ohm caused by reactivity and load unbalance.

Under asymmetrical (unbalanced) load the modulus of the reactive power is less than *Fryze* inactive power

(20) $$|Q| < Q_F, \qquad Q_F^2 = Q^2 + |\boldsymbol{D}|^2.$$

The power factor

(21) $$\lambda^2 = \lambda_P^2 = \frac{P^2}{S_B^2} = \frac{I_a^2}{I_a^2 + I_F^2}$$

is equal to unit solely when the active current is equal to the total current.

Three-phase three-wire system

Let us consider a three-phase, three-wire system with a triangle-connected linear load (△ -load)

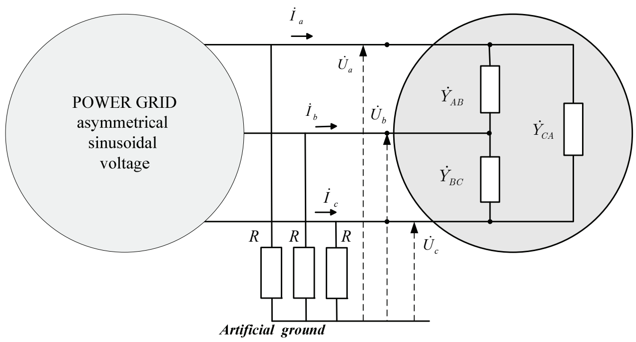

Fig. 1. Three-phase system with an unbalanced load

The current-voltage 3D-phasor relation is specified by the node matrix-vector equation

(22) $$\boldsymbol{I} = \mathcal{Y}\boldsymbol{U}.$$

Here the △ -load nodal admittance matrix

(23) $$\mathcal{Y} = \begin{bmatrix} \dot{Y}_{AB} + \dot{Y}_{CA} & -\dot{Y}_{AB} & -\dot{Y}_{CA} \\ -\dot{Y}_{AB} & \dot{Y}_{AB} + \dot{Y}_{CB} & -\dot{Y}_{CB} \\ -\dot{Y}_{CA} & -\dot{Y}_{CB} & \dot{Y}_{CA} + \dot{Y}_{CB} \end{bmatrix}$$

is defined by the equation

(24) $$\mathcal{Y} = \mathcal{M}\mathcal{Y}_{\triangle}\mathcal{M}^{\top},$$

where the diagonal interphase admittance matrix $\mathcal{Y}_{\triangle}$ is defined by the diagonal matrix

(25) $$\mathcal{Y}_{\triangle} = diag[\dot{Y}_{AB}, \dot{Y}_{BC}, \dot{Y}_{CA}].$$

$\mathcal{M}$ is the incidence matrix of the △ - load

(26) $$\mathcal{M} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

and $\mathcal{M}^{\top}$ stands for the transposed matrix of $\mathcal{M}$.

For a three-wire system any 3D-phasor $\boldsymbol{F} = (\dot{F}_a, \dot{F}_b.\dot{F}_c)^\top$ characterizing energy processes in the system is orthogonal to the zero–sequence ort $\boldsymbol{e_0} = (1,1,1)/\sqrt{3}$ because it satisfies the condition

(27) $$\dot{F}_0 = (\boldsymbol{F}, \boldsymbol{e}_0) = \boldsymbol{F}^\top \boldsymbol{e}_0 = (\dot{F}_a + \dot{F}_b + \dot{F}_c)/\sqrt{3} = 0 .$$

The respective energy process $\boldsymbol{f}(t) = \sqrt{2}\Re e[\boldsymbol{F}e^{j\omega t}]$ does not contain the zero component (the process is two-dimensional). For the current 3D-phasor $\boldsymbol{I}$ this condition is ensured by Kirchhoff's first law. For voltage 3D-phasor the condition (27) is ensured by selecting an artificial reference point (artificial ground [9]) $\dot{U}_{ref} = (\dot{U}_a + \dot{U}_b + \dot{U}_c)/3$, or by changing for the vector $\tilde{\boldsymbol{U}} = \boldsymbol{U} - \dot{U}_{ref}\boldsymbol{e}_0$ ($\tilde{\boldsymbol{U}}^\top \boldsymbol{e}_0 = 0$).

If 3D-phasor satisfies the condition (27), it is unambiguously decomposed by the orts

(28) $$\boldsymbol{e_1} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ \alpha^* \\ \alpha \end{pmatrix}, \quad \boldsymbol{e_2} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ \alpha \\ \alpha^* \end{pmatrix}$$

of positive and negative symmetrical sequences ($\alpha = e^{j2\pi/3}$, $1 + \alpha + \alpha^* = 0$, $\alpha^2 = \alpha^*$, $\alpha\alpha^* = 1$).
Thus the following implication is true

(29) $$(\boldsymbol{F}, \boldsymbol{e}_0) = 0 \quad \Rightarrow \quad \boldsymbol{F} = \dot{F}_1\boldsymbol{e_1} + \dot{F}_2\boldsymbol{e_2} .$$

Since orts (28) define orthonormal basis of symmetrical component

$$(\boldsymbol{e_k}, \boldsymbol{e_l}) = \boldsymbol{e_k}^\top (\boldsymbol{e_l})^* = \begin{cases} 1, k = l \\ 0, k \neq l \end{cases},$$

then

$$\dot{F}_k = (\boldsymbol{F}, \boldsymbol{e_k}) = \boldsymbol{F}^\top \boldsymbol{e}_k^* \ , \quad (k = 1,2).$$

The symmetrical basis (28) allows to determine the transformation matrix of symmetrical coordinates (positive and negative sequences) to the phase ones

(30) $$[\boldsymbol{e_1}\ \boldsymbol{e_2}] = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 \\ \alpha^* & \alpha \\ \alpha & \alpha^* \end{bmatrix} = \mathcal{F}_0$$

We denote for the 3D-vector (29) the column vector of its symmetrical coordinates

(31) $$\tilde{\boldsymbol{F}} = \left[\dot{F}_1\ \dot{F}_2\right]^\top .$$

Then expression (29) can be written as

(32) $$\boldsymbol{F} = [\boldsymbol{e_1}\ \boldsymbol{e_2}]\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}, \qquad \boldsymbol{F} = \mathcal{F}_0\tilde{\boldsymbol{F}} .$$

Hermitian - conjugate matrix

(33) $$\mathcal{F}_0^H = (\mathcal{F}_0^\top)^* = \begin{bmatrix} \boldsymbol{e_2}^\top \\ \boldsymbol{e_1}^\top \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix}$$

defines the inverse transform of the phase coordinates to the symmetrical ones.

(34) $$\tilde{\boldsymbol{F}} = \mathcal{F}_0^H \boldsymbol{F}$$

The matrix product (30) and (33) is equal to the identity matrix of second-order

(35) $$\mathcal{F}_0^H \cdot \mathcal{F}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \mathcal{F}_0^\top \cdot \mathcal{F}_0^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

We can check that if (29) is satisfied then $\mathcal{F}_0 \cdot \mathcal{F}_0^H \boldsymbol{F} = \boldsymbol{F}$ holds, and transforms (30) and (33) retain the inner product for such 3D-phasors.

Admittance matrix in symmetrical coordinates

From (34) it follows that for the voltage and current 3D-phasors the vector-matrix equations hold

(36) $$\tilde{\boldsymbol{I}} = \mathcal{F}_0^H \boldsymbol{I} , \qquad \tilde{\boldsymbol{U}} = \mathcal{F}_0^H \boldsymbol{U} .$$

The transformations chain of vector-matrix equation (22)

$$\mathcal{F}_0^H \boldsymbol{I} = \mathcal{F}_0^H \mathcal{Y}(\mathcal{F}_0\mathcal{F}_0^H)\boldsymbol{U} = \underbrace{\mathcal{F}_0^H \mathcal{Y}\mathcal{F}_0}_{\mathcal{H}}(\mathcal{F}_0^H \boldsymbol{U})$$

gives Ohm's law for △ -load in symmetrical coordinates

(37) $$\tilde{\boldsymbol{I}} = \mathcal{H}\tilde{\boldsymbol{U}} , \qquad \mathcal{H} = \mathcal{F}_0^H \mathcal{Y}\mathcal{F} .$$

In the symmetrical coordinates the coefficients of the nodal admittance matrix

(38) $$\mathcal{H} = \begin{bmatrix} \dot{Y}_{11} & \dot{Y}_{12} \\ \dot{Y}_{21} & \dot{Y}_{22} \end{bmatrix} = \begin{bmatrix} \boldsymbol{e_2}^\top \mathcal{Y}\boldsymbol{e_1} & \boldsymbol{e_2}^\top \mathcal{Y}\boldsymbol{e_2} \\ \boldsymbol{e_1}^\top \mathcal{Y}\boldsymbol{e_1} & \boldsymbol{e_1}^\top \mathcal{Y}\boldsymbol{e_2} \end{bmatrix}$$

can be obtained by direct calculus

$$Y_{11} = Y_{22} = Y_{AB} + Y_{BC} + Y_{CA}$$
$$\dot{Y}_{12} = -(\alpha\dot{Y}_{AB} + \dot{Y}_{BC} + \alpha^*\dot{Y}_{CA})$$
$$Y_{21} = -(\alpha^* Y_{AB} + Y_{BC} + \alpha Y_{CA}) .$$

Algebraic form of admittance $\dot{Y} = G + jB$ yields the next structure

(39) $$\mathcal{H} = \mathcal{H}' + j\mathcal{H}'' = \begin{bmatrix} G & \dot{C} \\ C^* & G \end{bmatrix} + j\begin{bmatrix} B & \dot{D} \\ D^* & B \end{bmatrix}$$

of the admittance matrix (38) of the positive and negative sequences. The coefficients

(40) $$G = G_{AB} + G_{BC} + G_{CA}$$
$$\dot{C} = e^{-j\pi/3}G_{AB} - G_{BC} + e^{j\pi/3}G_{CA}$$

of the matrix

(41) $$\mathcal{H}' = \begin{bmatrix} G & \dot{C} \\ C^* & G \end{bmatrix}$$

are conditioned by the active elements of the Δ - load.

The coefficients

(42)
$$B = B_{AB} + B_{BC} + B_{CA}$$
$$\dot{D} = e^{-j\pi/3}B_{AB} - B_{BC} + e^{j\pi/3}B_{CA}$$

of the matrix

(43)
$$\mathcal{H}'' = \begin{bmatrix} B & \dot{D} \\ D^{*} & B \end{bmatrix}$$

are conditioned by the reactive elements of the Δ - load.

Δ - Compensator with LC elements

To eliminate the inactive current from the circuit of the supplier (from the supply circuits) a compensator is set at the consumer connection point (Fig. 2).

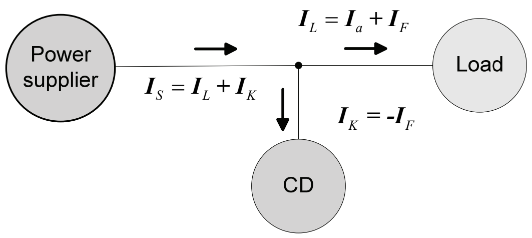

Figure. 2. Connection of the unbalance load with compensating device (CD)

The loading current $\boldsymbol{I}_L$, supply current $\boldsymbol{I}_S$ and compensator current $\boldsymbol{I}_K$ are connected by Kirchhoff's first law

(44)
$$\boldsymbol{I}_S = \boldsymbol{I}_L + \boldsymbol{I}_K .$$

From $\boldsymbol{I}_K = \boldsymbol{-I}_F$ it follows $\boldsymbol{I}_S = \boldsymbol{I}_a$. Since the $\Delta -$ compensator must not consume active energy, then the compensator circuitry must contain reactive (LC) elements alone. By (37) and (43) the symmetrical current components $\tilde{\boldsymbol{I}}_{\boldsymbol{K}} = (\dot{I}_{K1}, \dot{I}_{K2})^{\top}$ of $\Delta -$ compensator with a reactive LC - elements must satisfy the matrix-vector equation

(45)
$$\begin{bmatrix} \dot{I}_{K1} \\ \dot{I}_{K2} \end{bmatrix} = j \begin{bmatrix} B_K & \dot{D}_K \\ D_K^{*} & B_K \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} .$$

In equation (46) the current and voltage values are known and $\Delta -$ compensator matrix

(46)
$$\mathcal{B}_K = \begin{bmatrix} B_K & \dot{D}_K \\ D_K^{*} & B_K \end{bmatrix}$$

is not known. Since the value B_K is real, we have from the second equation (46)

(47)
$$I_{K2}^{*} = -jB_K U_2^{*} - j\dot{D}_K U_1^{*} .$$

The equality (47) leads to the following system of equations

(48)
$$\begin{bmatrix} \dot{\gamma}_1 & \dot{\gamma}_2 \\ \gamma_2^{*} & \gamma_1^{*} \end{bmatrix} \begin{bmatrix} B_K \\ \dot{D}_K \end{bmatrix} = \frac{j}{|\tilde{\boldsymbol{U}}|} \begin{bmatrix} -\dot{I}_{K1} \\ I_{K2}^{*} \end{bmatrix},$$

where we denote

$$\dot{\gamma}_2 = \dot{U}_2 / |\tilde{\boldsymbol{U}}|, \quad . \quad \dot{\gamma}_1 = \dot{U}_1 / |\tilde{\boldsymbol{U}}| .$$

Matrix of the normalized voltages

$$\mathcal{U} = \begin{bmatrix} \dot{\gamma}_1 & \dot{\gamma}_2 \\ \gamma_2^{*} & \gamma_1^{*} \end{bmatrix}$$

has the inverse matrix $\mathcal{U}^{-1}$. Multiplying at the left the equation (48) by a matrix $\mathcal{U}^{-1}$ we shall obtain

(49)
$$\begin{bmatrix} B_K \\ \dot{D}_K \end{bmatrix} = \frac{j\Delta}{|\tilde{\boldsymbol{U}}|} \begin{bmatrix} \gamma_1^{*} & -\dot{\gamma}_2 \\ -\gamma_2^{*} & \dot{\gamma}_1 \end{bmatrix} \begin{bmatrix} -\dot{I}_{K1} \\ I_{K2}^{*} \end{bmatrix},$$

where we denote $\Delta = (|\dot{\gamma}_1|^2 - |\dot{\gamma}_2|^2)^{-1}$.

Hence, B_K and $\dot{D}_K$ (D_K^{*}) are defined unambiguously

(50)
$$\begin{cases} B_K = -\dfrac{j\Delta}{|\tilde{\boldsymbol{U}}|}(\dot{I}_{K1}\gamma_1^{*} + I_{K2}^{*}\dot{\gamma}_2) \\ \dot{D}_K = \dfrac{j\Delta}{|\tilde{\boldsymbol{U}}|}(\dot{I}_{K1}\gamma_2^{*} + I_{K2}^{*}\dot{\gamma}_1) \\ D_K^{*} = -\dfrac{j\Delta}{|\tilde{\boldsymbol{U}}|}(I_{K1}^{*}\dot{\gamma}_2 + \dot{I}_{K2}\gamma_1^{*}) \end{cases} .$$

Denote the interphase susceptances vector of the $\Delta -$ compensator by $\boldsymbol{b}_{\boldsymbol{K}}^{\top} = \left(B_{AB}^{k}, B_{BC}^{k}\ B_{CA}^{k}\right)^{\top}$. The values (50) obtained above, B_K and $\dot{D}_K$ (D_K^{*}), are connected with the susceptances B_{AB}^{k}, B_{BC}^{k}, B_{CA}^{k} by the relations (42). This gives

(51)
$$\begin{cases} \sqrt{3}\boldsymbol{e}_{\boldsymbol{0}}^{\top}\boldsymbol{b}_{\boldsymbol{K}} = B_{AB}^{k} + B_{BC}^{k} + B_{CA}^{k} = B_K \\ e^{-j\pi/3}\sqrt{3}\boldsymbol{e}_{\boldsymbol{1}}^{\top}\boldsymbol{b}_{\boldsymbol{K}} = e^{-j\pi/3}B_{AB}^{k} - B_{BC}^{k} + e^{j\pi/3}B_{CA}^{k} = \dot{D}_K \\ e^{j\pi/3}\sqrt{3}\boldsymbol{e}_{\boldsymbol{2}}^{\top}\boldsymbol{b}_{\boldsymbol{K}} = e^{j\pi/3}B_{AB}^{k} - B_{BC}^{k} + e^{-j\pi/3}B_{CA}^{k} = D_K^{*} \end{cases}$$

Let us define the Fortescue transformation matrix from phase coordinates to the symmetrical ones (in the unitary form [12]) as

(52)
$$\mathcal{F} = \left[\boldsymbol{e}_{\boldsymbol{0}}\ \boldsymbol{e}_{\boldsymbol{1}}\ \boldsymbol{e}_{\boldsymbol{2}}\right] = \begin{bmatrix} \boldsymbol{e}_{\boldsymbol{0}}^{\top} \\ \boldsymbol{e}_{\boldsymbol{1}}^{\top} \\ \boldsymbol{e}_{\boldsymbol{2}}^{\top} \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^{*} & \alpha \\ 1 & \underbrace{\alpha \quad \alpha^{*}}_{\sqrt{3}\mathcal{F}_0} \end{bmatrix}$$

Matrix (52) is symmetrical and is obtained by adding column $\boldsymbol{e}_{\boldsymbol{0}}$ to the above matrix (30) on the left. Then the system of equations (51) can be written in vector-matrix form as

$$\sqrt{3}\mathcal{F}\boldsymbol{b}_{\boldsymbol{K}} = (B_K, e^{j\pi/3}\dot{D}_K, e^{-j\pi/3}D_K^{*})^{\top} .$$

Thus one arrives at the system of equations for determining the reactive elements of the $\Delta-$compensator for the given asymmetric voltage and the required compensating current

(53) $$\mathcal{F}\boldsymbol{b}_{\boldsymbol{K}} = \frac{j\Delta}{\sqrt{3}\,|\tilde{\boldsymbol{U}}|}\begin{bmatrix} -(\dot{I}_{K1}\gamma_1^* + I_{K2}^*\dot{\gamma}_2) \\ e^{j\pi/3}(\dot{I}_{K1}\gamma_2^* + I_{K2}^*\dot{\gamma}_1) \\ -e^{-j\pi/3}(I_{K1}^*\dot{\gamma}_2 + \dot{I}_{K2}\gamma_1^*) \end{bmatrix}$$

For Fortescue matrix (52) the inverse matrix is the complex conjugate matrix (the factor $1/\sqrt{3}$ is essential [12]).

(54) $$\mathcal{F}^{-1} = \mathcal{F}^* = \frac{1}{\sqrt{3}}\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^* & \alpha \\ 1 & \alpha & \alpha^* \end{bmatrix}}_{\sqrt{3}(\mathcal{F}_0^*)^\top}$$

Multiplying equation (53) on $\mathcal{F}^*$ at the left, we find unambiguously the interphase susceptances vector $\boldsymbol{b}_{\boldsymbol{K}}^\top = \left(B_{AB}^k, B_{BC}^k\ B_{CA}^k\right)^\top$ of the $\Delta-$compensator.

Simulative Examples

The computations were carried out in the MatCad environment. In the examples considered all the values are given in relative units $|\boldsymbol{U}| = 1$ pu.). The voltage 3D-phasor $\boldsymbol{U} = (0.632,\ 0.548e^{j234{,}8^o},\ 0.548e^{j125{,}2^o})^\top$ has symmetrical coordinates $\dot{U}_1 = 0.998$, $\dot{U}_2 = 0.1$, $\dot{U}_0 = 0$.

The voltage unbalance factor with respect to the negative sequence is equal to $k_{U2}=10\%$. The $\Delta-$load is specified by the interphase load admittance $\dot{Y}_{AB}, \dot{Y}_{BC}, \dot{Y}_{CA}$ (Table 1).

Table 1 Admittances of The $\Delta-$Load

№	$\dot{Y}_{AB}$	$\dot{Y}_{BC}$	$\dot{Y}_{CA}$
1	1	0	0
2	$1 - j4$	0	0
2	$0.4 - j0.1$	$0.7 - j0.3$	$0.1 - j0.2$

In all three examples the load is chosen to provide the transfer power with the same active power $P = 1.1$ pu. The interphase susceptances of the $\Delta-$compensator calculated according to (48-52) are given in Table 2.

Table 2. $\Delta-$Compensator Susceptances

№	jB_{AB}^k	jB_{BC}^k	jB_{CA}^k
1	$j0.047$	$j0.647$	$-j0.519$
2	$j4.047$	$j0.647$	$-j0.519$
3	$-j0.226$	$j0.494$	$j0.384$

After compensation in all three examples:

- additional losses from inactive current $\boldsymbol{I_F} = 0$ are completely eliminated and power factor is equal to unity $\lambda_P = 1$;
- Full losses are reduced in $|\boldsymbol{I}|^2 / |\boldsymbol{I_a}|^2$ times;
- Amplitudes of IP pulsation are reduced by $|\dot{N}| / |\dot{N}_a|$ times.

Here $\dot{N} = \boldsymbol{U}^\top\boldsymbol{I}$ and $\dot{N}_a = \boldsymbol{U}^\top\boldsymbol{I_a}$ are the complex amplitudes of power pulsation of the total (9) and active currents [16] .

Table 3. Simulation Resalts

	Before compensation			After compensation	
№	λ_P	$\lvert\boldsymbol{I}\rvert^2$	$\lvert\dot{N}\rvert$	$\lvert\boldsymbol{I_a}\rvert^2$	$\lvert\dot{N}_{a}\rvert$
1	0.741	2,22	1,104	1,208	0,218
2	0.18	37,74	4.554	1,208	0,218
3	0.785	2.023	0.414	1,234	0,22

Example 1. Modified Steinmetz symmetrizator-compensator

A one-phase active load $G_{AB} = 1$ is included between phases A and B. The interphase load admittances «load + compensator» ($\dot{Y}_{AB} = 1 + j0.047$, $\dot{Y}_{BC} = j0.647$, $\dot{Y}_{AC} = -j0.519$) differ from the admittances of the *Steinmetz circuit* under symmetrical voltage($\dot{Y}_{BC} = -\dot{Y}_{AC} = jG_{AB}/\sqrt{3} = j0.577$, $\dot{Y}_{AB} = G_{AB} = 1$).

Additional losses from the reactive and unbalanced currents are completely eliminated by CD. The pulsation amplitude IP is reduced by $|\dot{N}| / |\dot{N}_a| = 5{,}1$ times.

Example 2. Inductor

A one-phase load (inductor) is included between phase A and phase B. The inductor power factor is equal $\cos\varphi_{AB} = G_{AB} \big/ \sqrt{G_{AB}^2 + B_{AB}^2} = 0.243$. (The introduced circuit being a 3-phase load, the inductor power factor is calculated by (21) and is equal to $\lambda_P = 0.18$). In the second example the structural asymmetry is the same as in the first one and is compensated by the interphase admittances ($\dot{Y}_{AB} = j0.047$, $\dot{Y}_{BC} = j0.647$, $\dot{Y}_{AC} = -j0.519$). The additional inductive susceptance $jB_{AB} = -j4$ in arm AB is compensated by the additional capacity $jB_{AB}^k = j4$ of CD. The pulsation amplitude is reduced by $|\dot{N}| / |\dot{N}_a| = 20{,}93$ times.

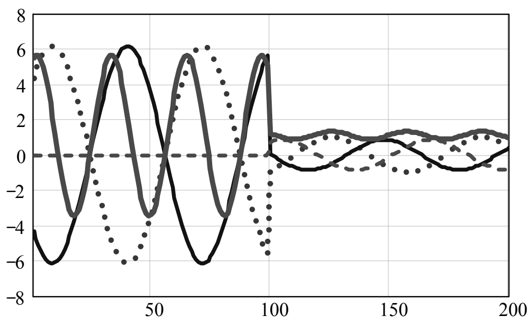

Fig. 3. Currents in phases and IP pulsation before and after compensation.

Fig. 3 demonstrates the changes in the supply current and IP pulsation after the power factor reaches the unity value $\lambda_P = 1$ for the inductor (Example 2).

Example 3. A typical three-phase unbalanced load.

Three single-phase active-inductive loads ($\cos\varphi_{AB} = 0.994$, $\cos\varphi_{BC} = 0.919$, $\cos\varphi_{AC} = 0.447$) are inserted in three arms and form three-phase unbalanced $\Delta -$load. A parallel-connected CD provides total compensation of the inactive current and reduces the pulsation amplitude by $|\dot{N}|/|\dot{N}_a| = 1{,}89$ times. Other examples of modeling are given in [19].

Conclusions

1. The paper disproves the impossibility of compensating the inactive current (reactive + unbalanced current) by means of a reactive element under asymmetrical sinusoidal voltage within the frames of the CPC Power Theory.
2. The proposed method of symmetrical basis for 3-wire systems with two-dimensional processes allows to obtain the transformation matrices (in an unitary form) between the symmetric and phase coordinates. These matrices can be naturally extended to Fortescue unitary matrices. Such transformations retain the inner product of 3D-phasors (in particular, SCP, PP and active power).
3. The above simulation results showed that the compensator we have worked out eliminates the IP pulsations caused by the loading asymmetry, but does not eliminate the IP pulsations caused by the asymmetry of the voltage.

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