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Mathematical modelling of matrix- reactance frequency converter described by nonstationary differential equations

***Abstract**. This paper introduces a method for calculating steady-state processes in three-phase matrix-reactance frequency converters (MRFC) that are described by nonstationary periodical differential equations. An approximation of the solution is based on the use Galerkin's method and an extension of differential equations of one variable of time to equations of two variables of time. The results of calculations for processes in a three-phase MRFC with buck-boost topology are obtained in the form of a double Fourier series and compared with the numerical method.*

Streszczenie.** Artykuł przedstawia metodę obliczania stanów ustalonych w trójfazowych matrycowo- reaktancyjnych przekształtnikach prądu przemiennego (MRFC), opisanych za pomocą niestacjonarnych równań różniczkowych. Aproksymacja rozwiązania jest otrzymana przy wykorzystaniu metody Galerkina i rozszerzeniu równań różniczkowych z przestrzeni jednej zmiennej czasu do dwóch zmiennych czasu. Rezultaty obliczeń przedstawione są na przykładzie trójfazowego przekształtnika MRFC o topologii typu buck-boost w formie podwójnych szeregów Fouriera i porównane z metodą numeryczną.**(Modelowanie matematyczne przekształtników matrycowo- reaktancyjnych opisywanych niestacjonarnymi równaniami różniczkowymi)

Keywords: AC/AC converters, matrix-reactance frequency converter, Galerkin's method, extension of differential equations.
Słowa kluczowe: przekształtnik prądu przemiennego, matrycowo- reaktancyjny przekształtnik prądu przemiennego, metoda Galerkina, rozszerzenie równań różniczkowych

Introduction

The subject of this article is a numerical method used to calculate steady-state processes in a three-phase matrix-reactance frequency converter. MRFCs based on a matrix-reactance chopper are used for both frequency and voltage transformations [1, 2]. These converters allow also the regulation of the magnitude of output voltages. The processes in a MRFC system are described by nonstationary differential equations with periodical coefficients

$$\frac{d\boldsymbol{X}(t)}{dt} = \boldsymbol{A}(t)X(t) + \boldsymbol{B}(t) \tag{1}$$

where $X(t)$ is a vector of state variables and $A(t)$, $B(t)$ are a matrix and a vector of the converter. These equations can be solved using an approximation by the numerical-analytic method. The search for solutions is limited to converting nonstationary differential equations into stationary equations and their solutions. One method using such a transformation presented in [3, 4] is based on the Lapunov transformation. Then, in order to calculate the convolutions of functions the Laplace transform is applied. This method can be used to find transient and steady- state processes.

A commonly used method for modeling MRFC is the averaged state-space method [5]. The method involves the generation of a replacement model of the circuit with averaged state variables. Other methods for solving nonstationary differential equations are Hill's method [6] and its modification, Taft's method [7]. In both methods, the solution can be obtained on the basis of infinite numbers of equations with infinite numbers of variables.

To solve the nonstationary differential equation it is also possible to use the Galerkin method [8]. This method approximates the solution using a finite number of basis and weight functions, in which the basis and weight functions for this method are the same. The method can be applied directly to find the solution without any additional transformation of equations.

The Galerkin method is used for analysis of steady-state processes in AC/AC converters [9]. The method is based on the extension of differential equations of one variable of time to equations of two variables of time [10].

The goal of this article is to present a method which allows the use of such equations to find steady-state processes in three-phase MRFCs. The solution is found by the Galerkin method with base functions described by a double Fourier series. The results of the calculations are verified by a numerical method.

Extension of differential equations

The extension of differential equations (1) for an additional independent variable of time is used for obtaining periodic solutions to nonstationary differential equations which contain different frequencies. As an example we will consider a simple circuit shown in Figure 1 with two independent supply voltages with periods T and Θ

$$e_1(t) = e_1(t+T),$$

$$e_2(t) = e_2(t+\Theta).$$

We suppose that the periods T and Θ are incommensurable.

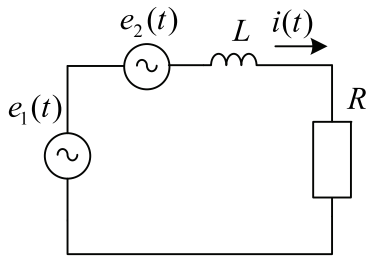

Fig. 1. Electric circuit with independent supply voltages

In such a circuit processes are described by the differential equation

$$L\frac{di(t)}{dt} + Ri(t) = e_1(t) + e_2(t). \tag{2}$$

It should be emphasized that the solution to (2), i.e., a current $i(t)$ is not periodical.

Let us use the superposition of solutions of two differential equations

$$L\frac{di_1(t)}{dt}+Ri_1(t)=e_1(t)\,,$$

$$L\frac{di_2(\tau)}{d\tau}+Ri_2(\tau)=e_2(\tau)\,.$$

The sum of the currents

$$i(t,\tau)=i_1(t)+i_2(\tau)\,.$$

will be periodical $i(t,\tau)=i(t+T,\tau+\Theta)$ with periods T and Θ.
We extend (2) by introducing the second independent variable of time

$$L\left(\frac{\partial i(t,\tau)}{\partial t}+\frac{\partial i(t,\tau)}{\partial \tau}\right)+Ri(t,\tau)=e_1(t)+e_2(\tau)\,.$$

In this case the steady-state solution will be periodical with periods T and Θ. The current $i(t)$ in the space of one variable of time can be obtained by simply equating $t=\tau$, i.e., $i(t)=i(t,t)$.

Such an approach is used for solving nonstationary differential equations with periodical coefficients. We extend the equations (1) for two independent variables of time as follows

$$\frac{\partial \boldsymbol{X}(t,\tau)}{\partial t}+\frac{\partial \boldsymbol{X}(t,\tau)}{\partial \tau}=\boldsymbol{A}(t,\tau)\boldsymbol{X}(t,\tau)+\boldsymbol{B}(t) \qquad (3)$$

where $X(t,\tau)$ is the vector of state variables.

In order to obtain the extended equation (3) it is necessary to replace the time variable t for τ in some places of the matrix $A(t)$ or vector $B(t)$ and introduce partial derivatives. In the case when the matrix $A(t)$ has different frequencies, we replace t for τ at one frequency and keep t at another frequency. If the matrix $A(t)$ and vector $B(t)$ have different frequencies then we replace t for τ in the matrix $A(t)$ or vector $B(t)$.

Converter general description

Let's consider the AC/AC three-phase MRFC with buck-boost topology. A control strategy in general form is illustrated in Fig. 2. In each sequence period T_{Seq} there are two time intervals, t_s and t_L. In the interval t_s the synchronous connected switches (SCS) are off, whereas matrix connected switches (MCS) function in accordance with a control strategy. In the interval MCS are off, whereas the SCS are on.

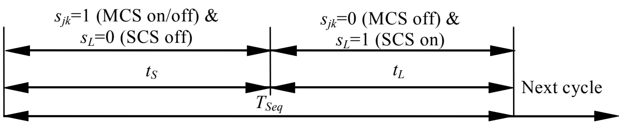

Figure 2. General description of the control strategy

The three-phase MRFC with buck-boost topology is shown in Fig. 3.
The switching functions are defined as

$$s_{jK}=\begin{cases}1, & switch\ S_{jK}\ closed\\ 0, & switch\ S_{jK}\ open\end{cases}$$

where $j=\{a,b,c\}$, $K=\{A,B,C\}$.

Voltage and current relations for the MCS are described as follows

$$\begin{bmatrix} i_A\\ i_B\\ i_C\end{bmatrix}=\begin{bmatrix} s_{aA} & s_{bA} & s_{cA}\\ s_{aB} & s_{bB} & s_{cB}\\ s_{aC} & s_{bC} & s_{cC}\end{bmatrix}\begin{bmatrix} i_a\\ i_b\\ i_c\end{bmatrix}=\boldsymbol{T}^T\begin{bmatrix} i_a\\ i_b\\ i_c\end{bmatrix},$$

$$\begin{bmatrix} u_a\\ u_b\\ u_c\end{bmatrix}=\begin{bmatrix} s_{aA} & s_{aB} & s_{aC}\\ s_{bA} & s_{bB} & s_{bC}\\ s_{cA} & s_{cB} & s_{cC}\end{bmatrix}\begin{bmatrix} u_A\\ u_B\\ u_C\end{bmatrix}=\boldsymbol{T}\begin{bmatrix} u_A\\ u_B\\ u_C\end{bmatrix},$$

where, T^T is the transposed instantaneous transfer matrix. In this converter the control signals presented in Fig. 2 are formed in line with classical control strategy [11]. The low-frequency transfer matrix takes the form

$$\boldsymbol{M}=\begin{bmatrix} d_{aA} & d_{aB} & d_{aC}\\ d_{bA} & d_{bB} & d_{bC}\\ d_{cA} & d_{cB} & d_{cC}\end{bmatrix}$$

where

$$d_{aA}=d_{bB}=d_{cC}=(1-D_S)(1+2q\cos(\omega_m t+\varphi))/3,$$
$$d_{aB}=d_{cA}=d_{bC}=(1-D_S)(1+2q\cos(\omega_m t+\varphi-2\pi/3))/3,$$
$$d_{aC}=d_{bA}=d_{cB}=(1-D_S)(1+2q\cos(\omega_m t+\varphi-4\pi/3))/3,$$

d_{jK} are the low frequency components of the MCS switching functions, $D_s=t_s/T_{Seq}$ is a sequence pulse duty factor, $\omega_m=\omega_L-\omega$, ω, ω_L are pulsations of the supply and load voltages respectively, q is a voltage gain.

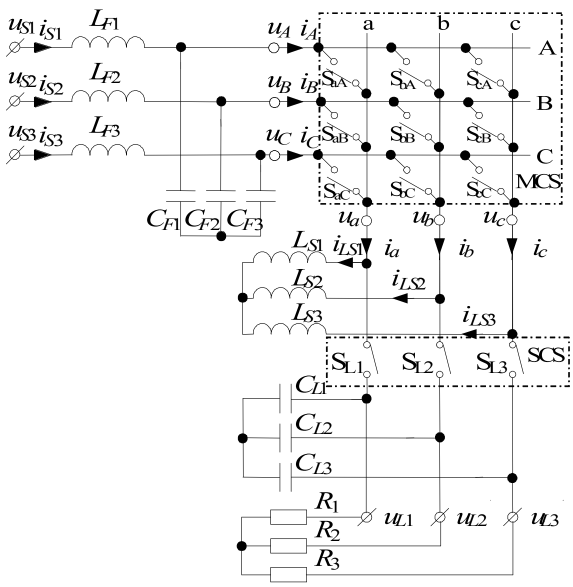

Fig. 3. MRFC based on buck-boost topology

Mathematical model

On the basis of the averaged state-space method [5] we can describe processes in such a system by the nonstationary differential equation (1), in which the vector of state variables has the form

$$X(t)=[i_{S1},i_{S2},i_{S3},i_{LS1},i_{LS2},i_{LS3},\\ u_{CF1},u_{CF2},u_{CF3},u_{L1},u_{L2},u_{L3}]^T$$

and the matrix $\boldsymbol{A(t)}$ and vector $\boldsymbol{B(t)}$ have the forms

$$A(t)=\begin{pmatrix}
\frac{-R_{F1}}{L_{F1}} & 0 & 0 & 0 & 0 & 0 & \frac{-1}{L_{F1}} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-R_{F2}}{L_{F2}} & 0 & 0 & 0 & 0 & 0 & \frac{-1}{L_{F2}} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{-R_{F3}}{L_{F3}} & 0 & 0 & 0 & 0 & 0 & \frac{-1}{L_{F3}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-R_{S1}}{L_{S1}} & 0 & 0 & \frac{d_{aA}}{L_{S1}} & \frac{d_{aB}}{L_{S1}} & \frac{d_{aC}}{L_{S1}} & \frac{1-D_{Seq}}{L_{S1}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-R_{S2}}{L_{S2}} & 0 & \frac{d_{bA}}{L_{S2}} & \frac{d_{bB}}{L_{S2}} & \frac{ds_{bC}}{L_{S2}} & 0 & \frac{1-D_{Seq}}{L_{S2}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-R_{S3}}{L_{S3}} & \frac{d_{cA}}{L_{S3}} & \frac{d_{cB}}{L_{S3}} & \frac{d_{cC}}{L_{S3}} & 0 & 0 & \frac{1-D_{Seq}}{L_{S3}} \\
\frac{1}{C_{F1}} & 0 & 0 & \frac{-d_{aA}}{C_{F1}} & \frac{-d_{bA}}{C_{F1}} & \frac{-d_{cA}}{C_{F1}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{C_{F2}} & 0 & \frac{-d_{aB}}{C_{F2}} & \frac{-d_{bB}}{C_{F2}} & \frac{-d_{cB}}{C_{F2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{C_{F3}} & \frac{-d_{aC}}{C_{F3}} & \frac{-d_{bC}}{C_{F3}} & \frac{-d_{cC}}{C_{F3}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{D_{Seq}-1}{C_{L1}} & 0 & 0 & 0 & 0 & 0 & \frac{-1}{R_1C_{L1}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{D_{Seq}-1}{C_{L2}} & 0 & 0 & 0 & 0 & 0 & \frac{-1}{R_2C_{L2}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{D_{Seq}-1}{C_{L3}} & 0 & 0 & 0 & 0 & 0 & \frac{-1}{R_3C_{L3}}
\end{pmatrix}$$

$$\boldsymbol{B}^T(t)=\begin{pmatrix} \frac{U_1\cos(\omega t)}{L_{F1}},\frac{U_2\cos(\omega t+2\pi/3)}{L_{F2}}, \\ \frac{U_3\cos(\omega t+4\pi/3)}{L_{F3}},0,0,0,0,0,0,0,0,0 \end{pmatrix}$$

where: R_{F1}, R_{F2}, R_{F3}, R_{S1}, R_{S2}, R_{S3} are resistances of inductors L_{F1}, L_{F2}, L_{F3} and L_{S1}, L_{S2}, L_{S3} respectively, U_1, U_2, U_3 are amplitudes of the supply source.

Method of calculation

The approximation of the solution by the Galerkin method is based on the introduction of the residue defined as a system of differential equations

(4) $$R_{X(t)}=\frac{d\boldsymbol{X}(t)}{dt}-\boldsymbol{A}(t)\boldsymbol{X}(t)-\boldsymbol{B}(t)$$

over the interval $0\le t\le T$.

The proposed method involves the extension of differential equations from (1) to (3). Then the residue (4) can be written as

(5) $$R_{X(t,\tau)}=\frac{\partial\boldsymbol{X}(t,\tau)}{\partial t}+\frac{\partial\boldsymbol{X}(t,\tau)}{\partial\tau}-\boldsymbol{A}(\tau)\boldsymbol{X}(t,\tau)-\boldsymbol{B}(t)$$

Taking base functions in the trigonometric form, we replace the vector $\boldsymbol{X}(t,\tau)$ in (5) by the vector $\boldsymbol{W}(t,\tau)$ whose components are

$$w^{(z)}(t,\tau)=\sum_{k=0}^{N}\sum_{n=0}^{N}[a_{kn}^{(z)}\phi_{kn}(t,\tau)+b_{kn}^{(z)}\psi_{kn}(t,\tau)+ \\ c_{kn}^{(z)}\theta_{kn}(t,\tau)+d_{kn}^{(z)}\xi_{kn}(t,\tau)]$$

where $z=1,..12$ correspond to the position of state variables in the vector $\boldsymbol{X}(t,\tau)$.

The method consists in determining the coefficients

$$a_{kn}^{(z)},b_{kn}^{(z)},c_{kn}^{(z)},d_{kn}^{(z)}$$

so that the residue (5) is minimum. For this purpose we multiply (5) by weight functions

$$\phi_{nk}(t,\tau)=\sin(n\omega t)\sin(k\sigma\tau),$$
$$\psi_{nk}(t,\tau)=\sin(n\omega t)\cos(k\sigma\tau),$$
$$\theta_{nk}(t,\tau)=\cos(n\omega t)\sin(k\sigma\tau),$$
$$\xi_{nk}(t,\tau)=\cos(n\omega t)\cos(k\sigma\tau)$$

and integrate the obtained expressions over the region defined by $0\le t\le T$, $0\le\tau\le\Theta$

$$\int_0^T\int_0^\Theta\phi_{kn}(t,\tau)R_{W(t,\tau)}d\tau dt=0,$$

$$\int_0^T\int_0^\Theta\psi_{kn}(t,\tau)R_{W(t,\tau)}d\tau dt=0,$$

$$\int_0^T\int_0^\Theta\theta_{kn}(t,\tau)R_{W(t,\tau)}d\tau dt=0,$$

$$\int_0^T \int_0^\Theta \xi_{kn}(t,\tau) R_{W(t,\tau)} d\tau dt = 0.$$

After the calculation of these integrals the result is presented in the form of linear equations. The solution of such equations is obtained in the form of the double Fourier series [12].

Calculation and simulation test results

The calculation of processes is realized with the help of the program Mathematica. The steady-state solution is obtained taking into account the following parameters of the circuit

$$R_{F1} = R_{F2} = R_{F3} = R_{S1} = R_{S2} = R_{S3} = 0{,}01\Omega,$$

$$R_1 = R_2 = R_3 = 10\Omega,\ U_1 = U_2 = U_3 = 310V,$$

$$C_{F1} = C_{F2} = C_{F3} = C_{L1} = C_{L2} = C_{L3} = 0{,}1\mu F,$$

$$L_{F1} = L_{F12} = L_{F3} = L_{L1} = L_{L2} = L_{L3} = 1\,mH$$

$$\omega_L = 2\pi / \Theta = 400\,rad/s,$$
$$\omega = 2\pi / T = 250\,rad/s,\quad D_S = 0{,}7,\ q = 0{,}4.$$

The steady-state solution to the equations (3) describing processes in the MRFC with buck-boost topology is obtained in the form of coefficients of the double Fourier series for the given parameters. The vector of steady-state variables is determined using bases and weight functions for $N=1$.

The exemplary steady-state current in the inductor L_{F1} is shown in Fig. 4, and the exemplary steady-state voltage across the capacitor C_{L1} is shown in Fig. 5, both in the space of two variables of time

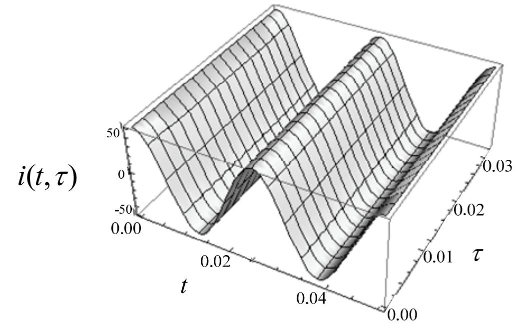

Fig.4. The steady-state current in the inductor L_{F1}

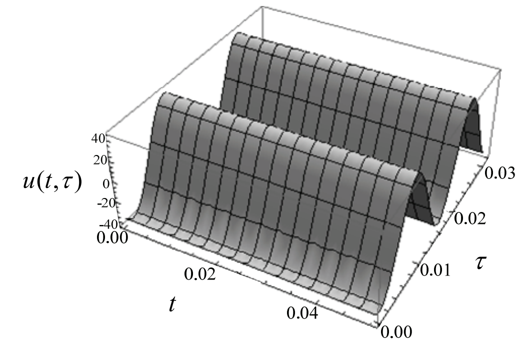

Fig. 5. The steady-state voltage across the capacitor C_{L1}

To verify the obtained results let us calculate the steady-state process for $t=\tau$ and then compare it with the results obtained by the numerical method. The exemplary currents in inductors are shown in Fig. 6 and 7 and voltages across capacitors in the steady-state obtained by the described method are shown in Fig. 8 and 9 respectively.

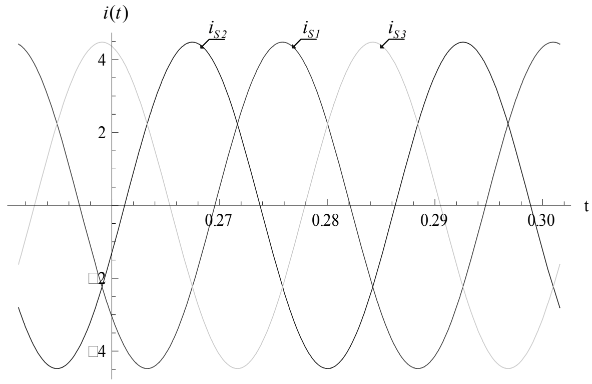

Fig. 6. Steady-state currents in the inductors L_{F1}, L_{F2}, L_{F3}

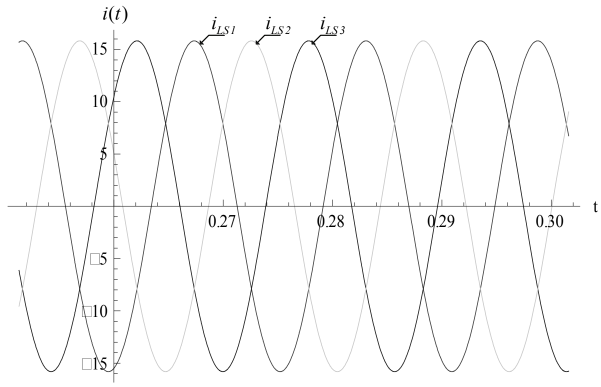

Fig. 7. Steady state currents in the inductors L_{L1}, L_{L2}, L_{L3}

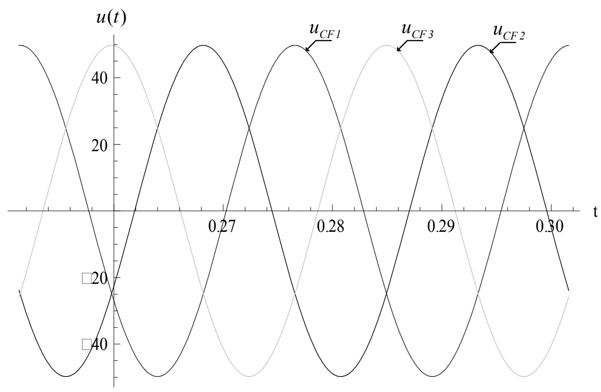

Fig. 8. Steady-state voltages across the capacitors C_{F1}, C_{F2}, C_{F3}

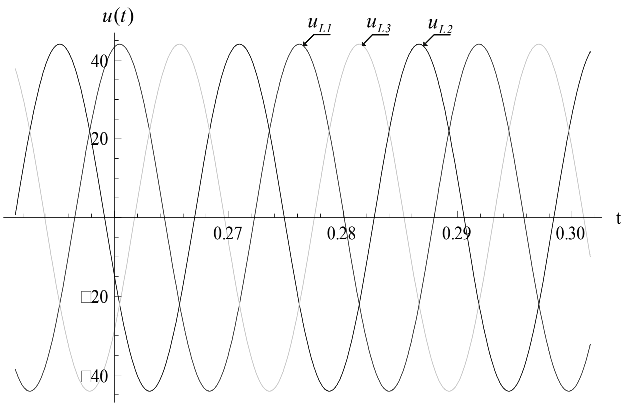

Fig. 9. Steady-state voltages across the capacitors C_{L1}, C_{L2}, C_{L3}

The same results are also obtained by the numerical method embedded in Mathematica.

It is known that the steady-state solutions obtained by the numerical method are determined only after the calculation of transient processes. Exemplaries of transient processes for current in the inductor i_{S1} and voltage across the capacitor u_{L1} obtained by using the numerical method and the steady-state process obtained by the proposed method are shown in Fig. 10 and 11.

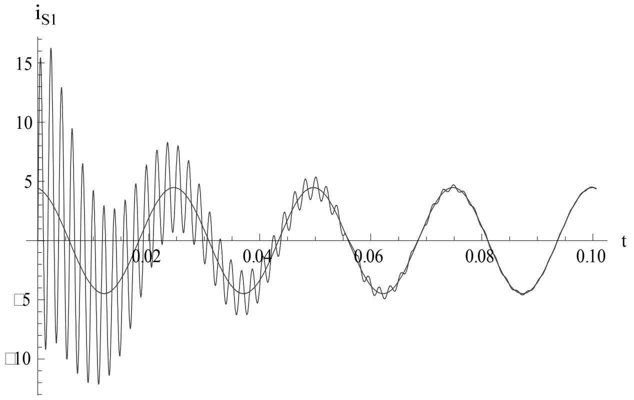

Fig. 10. Steady-state and transient exemplary currents in the inductor i_{S1}: red - the proposed method, blue – the numerical method.

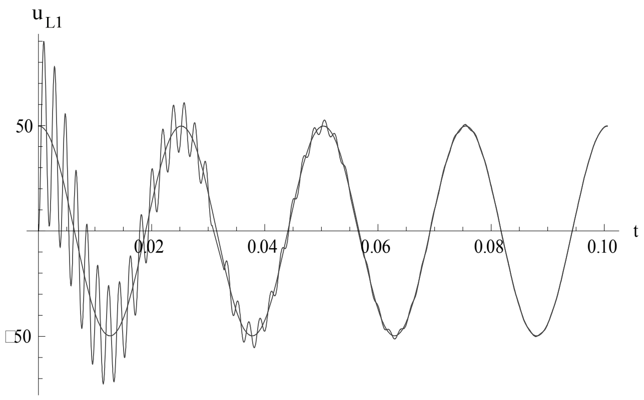

Fig. 11. Steady-state and transient exemplary voltages across capacitor u_{L1}: red – the proposed method; blue – the numerical method.

From these figures one can see that the transient process lasts some periods. In case of stiffness of differential equations, the calculations of steady-state processes might be became more difficult. Since the proposed method does not include the calculation of transition processes it allows minimization of calculation errors.

Conclusions

The considered method is intended for the calculation of steady-state processes in three-phase MRFCs. Differential equations have been extended by the introduction of the additional independent variable of time. A solution has been obtained by the Galerkin method assuming an aliquant relation between the periods of the supply source and the control signal. Calculations have been obtained for steady-state processes in the three-phase MRFC with buck-boost topology. The same results have also been obtained by using the numerical method.

It has been shown that with respect to numerical methods the calculation of transient processes is not necessary when the proposed method is employed.

This work was supported by the Polish Ministry of Science and Higher Education, Project No N N510 082138.

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